A Two-Stage Framework for Polygon Retrieval using Minimum Circular Error Bound

Lun Hsing Tung and Irwin King
{lhtung, king}@cse.cuhk.edu.hk

Department of Computer Science and Engineering,
The Chinese University of Hong Kong,
Shatin, New Territories, Hong Kong

Abstract

Shape matching is an important issue in pattern recognition and it has many real world applications. We have proposed a two-stage framework for polygon retrieval [12, 11] which incorporates both qualitative and quantitative measures of polygons in the first and second stage respectively. In this paper, we introduce an extension to our two-stage framework. We propose a new polygon matching technique using Circle Error Bound and describe how this technique works under translation and scaling of polygons. Base on this technique, we propose a new translation invariant similarity measure for polygons named Minimum Circular Error Bound, which can be used in the second stage of the two-stage framework. From observation, the Minimum Circular Error Bound method produces similarity ranking of polygons resembling human perception. We compare the Minimum Circular Error Bound method with the Hausdorff Distance method and demonstrate the advantages of our method.

1 Introduction

Shape matching and measuring similarity between shapes are important issues in pattern recognition. They can be applied in many applications, such as providing query-by-shape facility in image database systems and constructing hand writing recognition systems. In this paper, we focus on the matching of polygonal shapes instead of arbitrary shapes, since shapes are often represented by polygons and polygon approximation of shapes is acceptable in many applications.

In [12, 11], we proposed a two-stage framework for polygon retrieval in image databases. The first stage of the proposed framework uses the Binary Shape Descriptor (BSD) [2] technique to perform polygon classification, which serves as a mean for pruning the search space in order to speed up query processing. The second stage of the proposed framework uses any available polygon similarity measuring technique for quantitative measurement of the similarity between polygons. We proposed the Multi-Resolution Area Matching (MRAM) technique in [12, 11] as the technique to be incorporated at the second stage. In this paper, we propose an extension to the two-stage framework which allows systematic control on the degree of pruning the search space.

Considerable works have been carried out on the polygon matching problem. Most of these researches extract features from polygons and use these features as similarity measure [9, 6, 4, 10]. However, the similarity ranking of polygons produced by these methods may not coincide with human perception. In this paper, we propose a polygon matching technique using Circular Error Bound (CEB) which is based on an intuitive definition of polygon similarity. Based on the same idea, we propose a polygon similarity measure named Minimum Circular Error Bound (MCEB) which produces polygon rankings resembling human rankings. The MCEB method can be used at the second stage of the two-stage framework for shape matching.

The result of this work and the two-stage framework will be included in the *Montage* image database system [8], which is currently under development at the Chinese University of Hong Kong, for providing query-by-shape facility. The *Montage* system is an image database system designed for the fashion, textile, and clothing industry in Hong Kong. It supports feature based retrieval by color histogram, color sketch, shape, and texture.

This paper is organized as follows. We present the extension to the two-stage framework in Section 2. We propose the polygon matching technique using Circular Error Bound in Section 3. In Section 4, we present the Minimum Circular Error Bound similarity measure for polygons. The experimental results of our work will be discussed in Section 5. Conclusion is made in Section 6.

2 Extension to the Two-Stage Framework

In [12, 11], we proposed a two-stage framework for polygon retrieval in image databases. The first stage of the proposed framework uses the Binary Shape Descriptor (BSD) [2] technique to perform polygon classification. The second stage uses any available polygon similarity measuring technique for quantitative measurement of the similarity between polygons and ranks polygons by the order of the similarity between them and a particular target polygon.

A BSD is simply a binary string recording the convexities and concavities of the vertices of a polygon:

Definition 1 *Let '0' denotes a convex vertex (interior angle less than π) and '1' denotes a concave vertex (interior angle larger than π). A Binary String Descriptor (BSD) is a string $\{0,1\}^n$, where n is the number of vertices of the polygon.*

The idea of Standardized Binary Shape Descriptor (SBSD) [2, 12, 11] is developed in order to solve the problem that a polygon may have different BSDs when the BSDs are recorded starting at different vertices:

Definition 2 *Given a BSD $B = \{0,1\}^n$, a rotated BSD B_i, for $1 \leq i \leq n$, is another BSD resulted by rotating the bits of B such that the ith MSB (Most Significant Bit) of B becomes the MSB of B_i. Let $M(B_i)$ denotes the magnitude of B_i regarding it as a binary integer. The standardized Binary String Descriptor (SBSD) of B is B_j such that $M(B_j) = \min_i M(B_i), 1 \leq i \leq n$.*

The first stage of the two-stage framework serves as a mean for pruning the search space in order to reduce the number of polygons needed to be compared to the target polygon. The strategy of this stage that we have proposed is as follows:

0. The SBSD of a polygon is computed and stored in the database along with it when it is added to the database.
1. When a query is initiated, the SBSD of the target polygon is computed and only the polygons inside the database having exactly the same SBSD as the target polygon are selected.
2. A similarity ranking is performed on the selected polygons based on the degree of their similarity to the target polygon.

Step 0 is the database population process while step 1 and step 2 are the first stage and the second stage of the two-stage framework respectively.

The proposed method may fail to produce good matching results. In Figure 1, there are three polygons named P, Q, and R. Using polygon P as the target polygon, the proposed method will not be able to produce the result that polygon Q is more similar to polygon P than polygon R does. It is because polygon Q is not in the equivalent class as polygon P so it will not be selected at the first stage. On the other hand, polygon R is in the same equivalent class as polygon P so it is selected for the second stage matching. Yet, polygon

Q appears to be more similar to polygon P than polygon R to polygon P. To solve this problem, we propose the following extension to the two-stage framework. The step 1 of the original algorithm is changed to:

1. When a query is initiated, the SBSD of the target polygon is computed. All polygons inside the database having a SBSD within a user specified Hamming Distance to the SBSD of the target polygon are selected.

This extension provides a systematic way for controlling the degree of pruning database entries. Small Hamming Distance value has larger pruning effect but with higher risk of producing worse matching results. On the other hand, large Hamming Distance value may produce better matching result but will result in inefficiency since a lot of polygons will be selected for second stage matching.

For example, if user specifies a Hamming Distance of 1, both polygon Q and polygon R in Figure 1 will be selected for second stage matching so polygon Q will have the chance to be compare with polygon P. Whether polygon Q is said to be more similar to polygon P or not will still depends on the polygon similarity measuring method used at the second stage.

Let's use polygons with 5 sides as another example. For polygons with 5 sides, there are 4 possible SBSDs (00000, 00001, 00011, 00101). If the target polygon has the SBSD of 00001 and user specifies a Hamming Distance of 0, then only polygons in the database having SBSD of 00001 will be selected for second stage matching. If user specifies a Hamming Distance of 1, then polygons in the database having SBSD of 000000, 00001 or 00011 will be selected for second stage matching.

3 Polygon Matching using Circular Error Bound

In this section, we propose a polygon matching technique based on an intuitive human definition of polygon resemblance. The intuitive definition of similar polygons is as follows. If two polygons are similar (or matched), then each vertex of one polygon is close to its corresponding vertex of another polygon when the two polygons are overlapped. The correspondence between vertices is an one-to-one mapping. Therefore, the definition and the technique we proposed only work on polygons that have the same number of vertices. Before the two polygons are overlapped, translation, scaling and rotation are allowed to be performed on the polygons.

Definition 3 *A polygon P is represented by an ordered list of vertex coordinates, that is, $P = (V_1, V_2, \ldots, V_n)$, where $V_i \in \mathbb{R}^2$ and n is the number of vertices of polygon P.*

Definition 4 *A transformation T is a vector, i.e. $T = (t_x, t_y, s_x, s_y, \theta)$ where t_x is translation in X direction, t_y is translation in Y direction, s_x is the scaling in X direction, s_y is the scaling in Y direction and θ is the rotation about the origin. $T(Q)$ denotes the polygon (or vertex) obtained by applying T to Q.*

Definition 5 *Given a tolerance vector $E = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$, Q is said to be matched with P if there exists a transformation T such that $Q' = T(Q) = (U_i', U_2', \ldots, U_n')$ and $\forall_{1 \leq i \leq n} \|V_i - U_i'\| \leq \epsilon_i$, where $\| \cdot \|$ denotes the Euclidean norm* (Figure 2).

Definition 6 *Given V_i, ϵ_i and U_i, the ith* Circular Error Bound (**CEB**) *, C_i, is a circle with ϵ_i as its radius and $(V_i - U_i)$ as its center.*

Note that Definition 5 assumes we already know the pairing of vertices between the two polygons, i.e. V_i should match U_i. Figure 2 illustrates the idea of Definition 5.

The polygon matching task is formulated as follows:

> "Given two polygons P and Q with a tolerance vector E, the task is to determine whether a transformation T exists such that Q is said to be matched with P under Definition 5."

By Definition 5, the transformation T is an arbitrary tuple $(t_x, t_y, s_x, s_y, \theta)$. However, in nowadays applications, the transformations in polygon matching task are often restricted to some special cases, for example, translation and (or) scaling only. With restricted transformations, we have efficient solutions for the polygon matching task. In the following sections, we will present the solution for the polygon matching task when (1) only translations are allowed, (2) only translations and uniform scaling in X, Y direction are allowed, and (3) only translations and independent scaling in X, Y direction are allowed.

3.1 Translation

Assume that the transformation T in Definition 5 is restricted to $T = (t_x, t_y, 1, 1, 0)$ only.

Claim 1 *Given $P = (V_1, V_2, \ldots, V_n)$, $Q = (U_1, U_2, \ldots, U_n)$ and $E = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$, if the n Circular Error Bounds $C_1, C_2, \ldots, C_n$ of P and Q have common intersection, then Q is matched with P.*

Proof: Assuming $V_i = (a_i, b_i)$ and $U_i = (c_i, d_i)$, by Definition 6, Circular Error Bound C_i is a circle with ϵ_i as its radius and $(a_i - c_i, b_i - d_i)$ as its center. If $C_1, C_2, \ldots, C_n$ have common intersection, then for any point (t_x, t_y) in the common intersection, the distance between this point and the center of any C_i is larger or equal to the radius of C_i. Figure 3 illustrates this idea when both P and Q are triangles. Thus, $\forall_{1 \leq i \leq n}$

$$[(a_i - c_i) - t_x]^2 + [(b_i - d_i) - t_y]^2 \leq {\epsilon_i}^2 \tag{1}$$

Re-arranging Equation (1), we have

$$[a_i - (c_i + t_x)]^2 + [b_i - (d_i + t_y)]^2 \leq {\epsilon_i}^2 \tag{2}$$

which is equivalent to $\|V_i - U_i'\| \leq \epsilon_i$ where $U_i' = T(U_i)$ and $T = (t_x, t_y, 1, 1, 0)$. By Definition 5, Q is matched with P. ■

3.2 Translation and uniform scaling in X, Y direction

Assume that the transformation T in Definition 5 is restricted to $T = \langle t_x, t_y, s, s, 0 \rangle$, i.e. only translation and uniform scaling in X, Y direction are allowed.

Let $U_i = (c_i, d_i)$ and apply the scaling transformation $S = (0, 0, s, s, 0)$ to Q, we have $U_i' = S(U_i) = (sc_i, sd_i)$. Thus, Circular Error Bound C_i of P and Q', where $V_i = (a_i, b_i)$, is a circle with ϵ_i as its radius and $(a_i - sc_i, b_i - sd_i)$ as its center.

Two Circular Error Bounds C_i and C_j intersect each other if and only if

$$[(a_i - sc_i) - (a_j - sc_j)]^2 + [(b_i - sd_i) - (b_j - sd_j)]^2 \leq (\epsilon_i + \epsilon_j)^2 \tag{3}$$

Re-arranging Equation (3), we have

$$\begin{aligned} [(c_i - c_j)^2 + (d_i - d_j)^2]s^2 - 2[(a_i - a_j)(c_i - c_j) + (b_i - b_j)(d_i - d_j)]s \\ + [(c_i - c_j)^2 + (d_i - d_j)^2 - (\epsilon_i + \epsilon_j)^2] \leq 0 \end{aligned} \tag{4}$$

Solving Equation (4), we get a range, $\mathbb{S}_{ij}$, for s that the inequality holds (Figure 4).

Claim 2 *If $\bigcap_{1 \leq i,j \leq n} \mathbb{S}_{ij} \neq \emptyset$, then Q is matched with P.*

Proof: If $\bigcap_{1 \leq i,j \leq n} \mathbb{S}_{ij} \neq \emptyset$, then $\exists S = (0, 0, s, s, 0) \in \bigcap_{1 \leq i,j, \leq n} \mathbb{S}_{ij}$ such that Circular Error Bounds $C_1, C_2, \ldots, C_n$ of P and Q' have common intersection, where $Q' = S(Q)$. By Claim 1, Q' is matched with P. Thus, $\exists T = (t_x, t_y, 1, 1, 0)$ such that $\forall_{1 \leq i \leq n} \|V_i - U_i''\| \leq \epsilon_i$ where $U_i'' = T(U_i')$. Therefore, $\exists T' = T \circ S = (t_x, t_y, s, s, 0)$ such that $\forall_{1 \leq i \leq n} \|V_i - U_i''\| \leq \epsilon_i$ where $U_i'' = T'(U_i)$. By Definition 5, Q is matched with P. ■

3.3 Translation and independent scaling X, Y direction

Assume that the transformation T in Definition 5 is restricted to $T = \langle t_x, t_y, s_x, s_y, 0\rangle$, i.e. independent scaling in X, Y direction as well as translation are allowed.

Let $U_i = (c_i, d_i)$ and apply the scaling transformation $S = (0, 0, s_x, s_y, 0)$ to Q, we have $U_i' = S(U_i) = (s_x c_i, s_y d_i)$. Thus, Circular Error Bound C_i of P and Q', where $V_i = (a_i, b_i)$, is a circle with ϵ_i as its radius and $(a_i - s_x c_i, b_i - s_y d_i)$ as its center.

Two Circular Error Bounds C_i and C_j intersect each other if and only if

$$[(a_i - s_x c_i) - (a_j - s_x c_j)]^2 + [(b_i - s_y d_i) - (b_j - s_y d_j)]^2 \leq (\epsilon_i + \epsilon_j)^2 \tag{5}$$

Re-arranging Equation (5), we have

$$[(a_i - a_j) - (c_i - c_j)s_x]^2 + [(b_i - b_j) - (d_i - d_j)s_y]^2 \leq (\epsilon_i + \epsilon_j)^2 \tag{6}$$

Equation (6) defines an ellipse, $\mathbb{E}_{ij}$, on the s_x-s_y plane. A point (s_x, s_y) in $\mathbb{E}_{ij}$ defines a transformation $S = (0, 0, s_x, s_y, 0)$ such that when S is applied to Q, the Circular Error Bounds C_i and C_j, of $S(Q)$ and P, intersect each other (Figure 5).

Claim 3 *If* $\forall_{1 \leq i,j \leq n} \mathbb{E}_{ij}$ *have common intersection, then* Q *is matched with* P.

Proof: If $\forall_{1 \leq i,j \leq n} \mathbb{E}_{ij}$ have common intersection, then for any point (s_x, s_y) in the common intersection, Circular Error Bounds $C_1, C_2, \ldots, C_n$ of of P and Q', intersect each other, where $Q' = S(Q)$ and $S = (0, 0, s_x, s_y, 0)$. By Claim 1, Q' is matched with P. Thus, $\exists T = (t_x, t_y, 1, 1, 0)$ such that $\forall_{1 \leq i \leq n}, \|V_i - U_i''\| \leq \epsilon_i$ where $U_i'' = T(U_i')$. Therefore, $\exists T' = T \circ S = (t_x, t_y, s_x, s_y, 0)$ such that $\forall_{1 \leq i \leq n}, \|V_i - U_i''\| \leq \epsilon_i$ where $U_i'' = T'(U_i)$. By Definition 5, Q is matched with P. ■

4 Minimum Circular Error Bound

The results presented in Section 3.1, 3.2, and 3.3 only deal with queries of whether a polygon Q is matched with another polygon P subject to some tolerances (the tolerance vector E) and when the transformation T has different restrictions. It is also useful to find out how similar a polygon Q is comparing to another polygon P. For example, we may want to rank a list of polygons according to the similarity between these polygons and a target polygon. We propose a translation invariant similarity measure of polygons named **Minimum Circular Error Bound (MCEB)** based on the Circular Error Bound technique we described above.

Definition 7 *The Minimum Circular Error Bound, $\xi \in \mathbb{R}$, of a polygon $Q = (U_1, U_2, \ldots, U_n)$ comparing to another polygon $P = (V_1, V_2, \ldots, V_n)$ is defined as*

$$\xi = \min_{\forall_{t_x,t_y} T=(t_x,t_y)} \max_{1 \le i \le n} \|V_i - T(U_i)\|$$

ξ can be calculated as follows. Let $V_i = (a_i, b_i)$ and $U_i = (c_i, d_i)$. Further assume that the tolerance vector $E = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$ where $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n$. The Circular Error Bound C_i is a circle with ϵ_i as its radius and $(a_i - c_i, b_i - d_i)$ as its center. If two Circular Error Bounds C_i and C_j intersect each other, we have

$$[(a_i - c_i) - (a_j - c_j)]^2 + [(b_i - d_i) - (b_j - d_j)]^2 \le (\epsilon_i + \epsilon_j)^2 \tag{7}$$

Since $\epsilon_i = \epsilon_j$, we denote the value of ϵ_i and ϵ_j as ϵ_{ij}. The minimal value of ϵ_{ij} that Equation (7) holds is

$$\epsilon_{ij} = \frac{1}{2}\sqrt{[(a_i - c_i) - (a_j - c_j)]^2 + [(b_i - d_i) - (b_j - d_j)]^2}$$

The MCEB of the two polygons Q and P is

$$\xi = \max_{1 \le i,j \le n} \epsilon_{ij}$$

such that for $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n \ge \xi$, $\forall_{1 \le i,j \le n}$ C_i and C_j intersect each other. That is, for $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n \ge \xi$, Circular Error Bounds $C_1, C_2, \ldots, C_n$ of Q and P have common intersection and Q is matched with P under Claim 1.

5 Experimental Results

In this section, we compare the proposed MCEB method with the Hausdorff Distance method [3, 5].

Definition 8 *Given two finite point sets $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_m\}$, the Hausdorff Distance is defined as*

$$H(A, B) = \max(h(A, B), h(B, A))$$

where

$$h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\|$$

and $\|\cdot\|$ is some underlying norm on the points of A and B.

We choose Hausdorff Distance method for comparison since both methods measure polygon similarity based on the distance between polygon vertices. We will compare their similarity ranking results as well as the running time complexity.

5.1 Similarity Ranking

MCEB is designed for producing rankings of polygons resembling human rankings. In this session, we compare the polygon rankings produced by the MCEB and the Hausdorff Distance method. The experiment is conducted as follows. In each experiment, two polygons are selected for generating the input polygons. For example, Figure 6(a) and Figure 6(b) show the two polygons used in one experiment where Figure 6(a) is used as the first polygon and Figure 6(b) is used as the last one. By interpolating these two polygons, 48 intermediate polygons are generated, which gives us 50 polygons in total.

Using this method, we obtain a list of polygons, ranked by their relative similarity to the first polygon, which resembles human ranking. We then rank these 50 polygons using the MCEB method and the Hausdorff Distance method accordingly with the first polygon being the target polygon. We use the number of polygons, having different relative ranking as the original list, to measure the quality of the ranking produced. A small number indicates a good ranking which means that the ranking produced is similar to the original list, and human visual ranking as well.

Figure 7(a) and Figure 7(b) show the rankings produced by the MCEB method and the Hausdorff Distance method using Figure 6(a) and Figure 6(b) as the input data. The quality measure of the two rankings are 0 and 9 respectively. Figure 8(a) and Figure 8(b) show the rankings produced by the MCEB method and the Hausdorff Distance method using Figure 6(c) and Figure 6(d) as the input data. The quality measure of the two rankings are 0 and 13 respectively. In these two examples, the MCEB method produces better rankings than the Hausdorff Distance method. Note that the MCEB method succeeds in producing rankings resembling human visual ranking.

5.2 Running Time Complexity

The computational complexity of the Hausdorff Distance is $O(n^2)$ and it is $O(n^2)$ for the MCEB method, if the correspondence of vertices is known, or $O(n^3)$ if the correspondence of vertices is unknown since we have to exhaust the n possible correspondences in order to find out the overall MCEB for a n-gon. However, note that the MCEB method gives similarity measure between two polygons under the optimal translation, but the Hausdorff Distance method does not. We may want to use the Hausdorff Distance under optimal translation for ranking polygons instead of the original Hausdorff Distance in order to produce better ranking of polygons. The computational complexity for optimal Hausdorff Distance under translation is $O(n^4 \log^3(n^2))$ [1], which is much larger than the corresponding computational complexity of the MCEB method.

Table 1 shows the average query processing time of the MCEB method, the Hausdorff Distance method and the MRAM method we proposed in [12, 11]. The experiments are conducted using the simple system we described in [12]. There are 9000 polygons in each testing database and each database is consisted of polygons with a specific number of sides, from 3 to 8. The table shows that MCEB has an average query processing time that is less than the average query processing time of Hausdorff Distance method, while the MRAM has the smallest average query processing time among the three methods.

6 Conclusion

In this paper, we introduce an extension to the two-stage framework for polygon retrieval we proposed. We propose a new polygon matching technique using Circular Error Bound (CEB) method which is based on an intuitive definition similar to human concept. We give the formal definition of this idea and show how it works for polygon matching problem under translation and scaling of polygons. Based on CEB, we propose a similarity measure of polygons, which can be employed at the second stage of the two-stage framework, named Minimum Circular Error Bound (MCEB). We find that the MCEB method gives rankings of polygons similar to human rankings and it is more efficient than the Hausdorff Distance method.

Right now, the CEB method only handles polygon matching under translation and scaling. A natural extension to the CEB method is to incorporate rotation. Another possible extension to our work is to enhance the translation invariant MCEB method such that it is scaling invariant as well.

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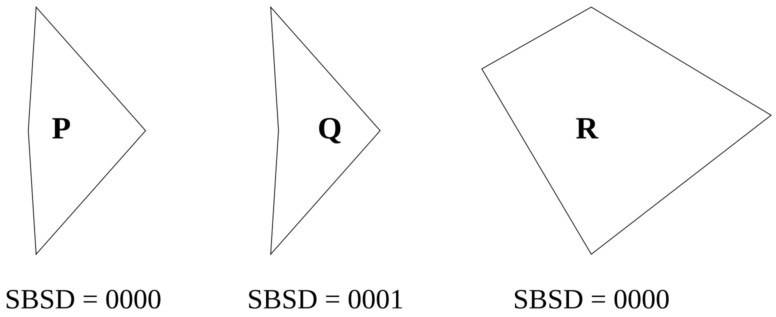


Figure 1: The problem of the two-stage framework

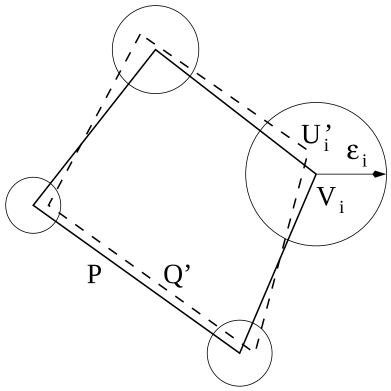


Figure 2: Polygon matching under Definition 5

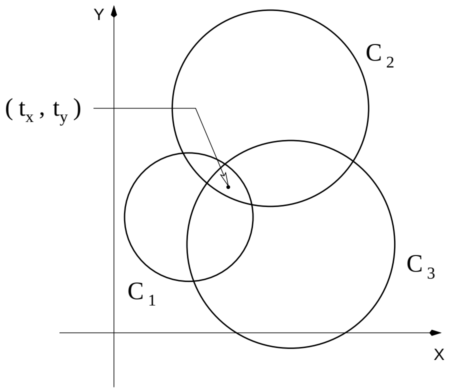


Figure 3: Intersection of Circular Error Bounds

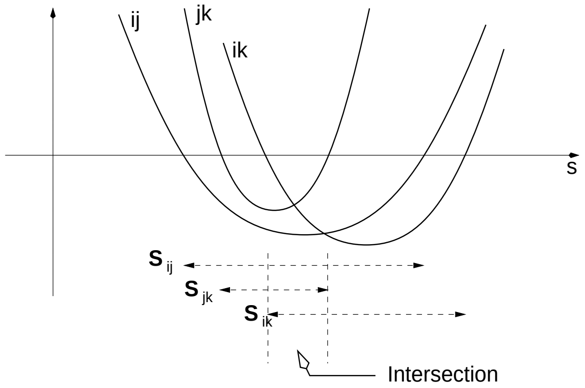


Figure 4: $\mathbb{S}_{ij}$ and its intersection

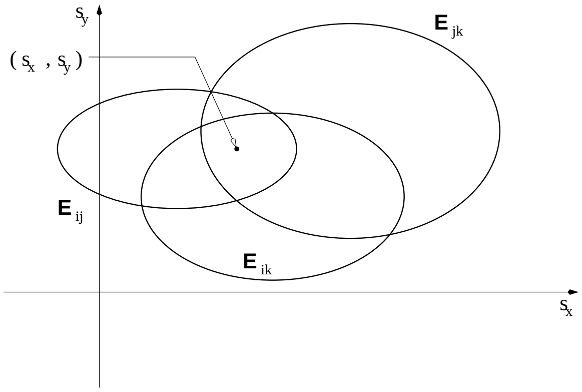
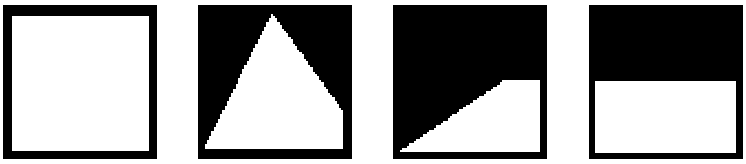
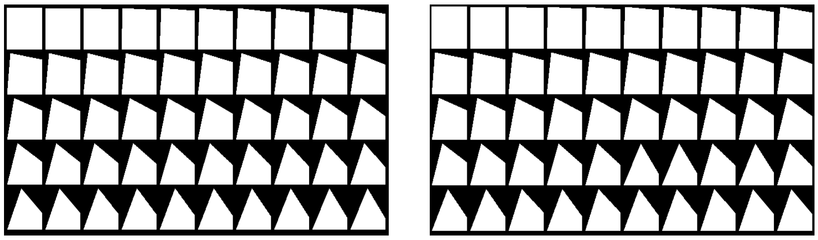


Figure 5: $\mathbb{E}_{ij}$ and its intersection



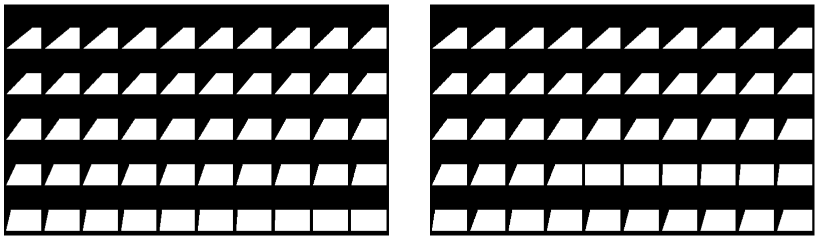
(a) (b) (c) (d)

Figure 6: Similar ranking experiment data



(a) (b)

Figure 7: Result of similar ranking experiment 1



(a) (b)

Figure 8: Result of similar ranking experiment 2

Table 1: Experiment results

N-gon	**Number of polygons**	**MRAM**	**MCEB**	**Hausdorff Distance**
3	9000	2.04 sec	3.24 sec	3.40 sec
4	9000	1.97 sec	2.40 sec	2.62 sec
5	9000	1.74 sec	2.00 sec	2.21 sec
6	9000	1.64 sec	1.78 sec	1.87 sec
7	9000	1.62 sec	1.68 sec	1.72 sec
8	9000	1.63 sec	1.67 sec	1.68 sec