

CENG4480

Lecture 03: Operational Amplifier – 2

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香港中文大學

The Chinese University of Hong Kong

Overview

Preliminaries

Integrator & Differentiator

Filters



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Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

- ▶ real component
- ▶ imaginary component
- ▶ magnitude

$$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$



Prove:

$$\left| \frac{1}{1+ja} \right| = \frac{1}{\sqrt{1+a^2}}$$



Prove:

$$\left| \frac{1}{1+ja} \right| = \frac{1}{\sqrt{1+a^2}}$$

Tips:

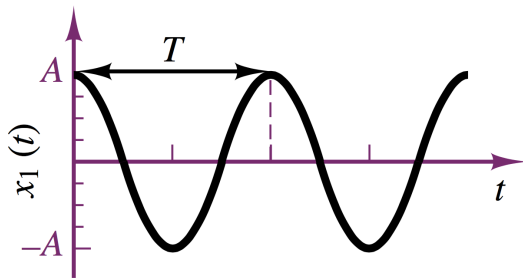
$$\begin{aligned} \frac{1}{1+ja} &= \frac{1}{1+ja} \cdot \frac{1-ja}{1-ja} \\ &= \frac{1-ja}{1+a^2} \\ &= \frac{1}{1+a^2} - \frac{ja}{1+a^2} \end{aligned}$$



Sinusoidal Signal

$$x(t) = A\cos(\omega t + \phi)$$

- ▶ Periodic signals
- ▶ A : amplitude
- ▶ ω : radian frequency
- ▶ ϕ : phase

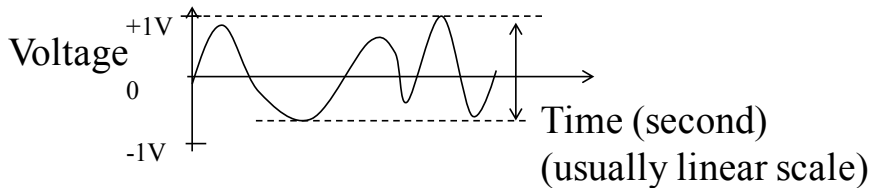


Time Domain

- ▶ Voltage gain against time

For sinusoidal signal:

$$v(t) = A\cos(\omega t + \phi)$$

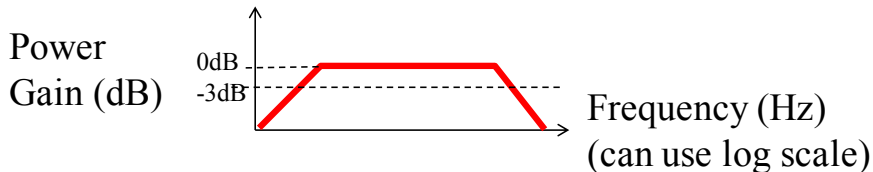


Frequency Domain

- ▶ Voltage gain against frequency

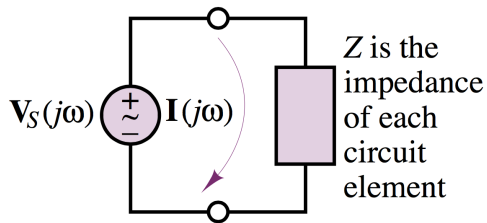
For sinusoidal signal:

$$\mathbf{V}(j\omega) = Ae^{j\phi} = A\angle\phi = A\cos\phi + jA\sin\phi$$



Impedance

A complex resistance or *frequency-dependent resistance*. That is, as resistors whose resistance is a function of the frequency of the sinusoidal excitation.



AC circuits in
phasor/impedance form

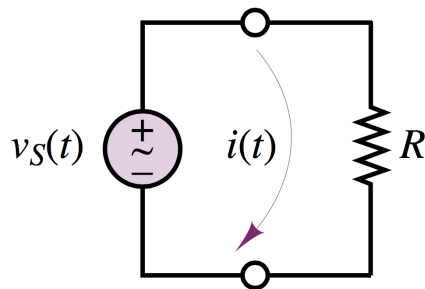


Resistor Impedance

Assume source voltage $v = A \cos(\omega t)$, then

▶ $\mathbf{V}(j\omega) = A\angle 0$

▶ $\mathbf{I}(j\omega) = \frac{A}{R}\angle 0$

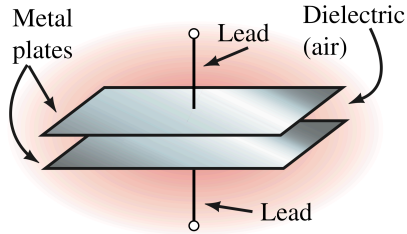


Impedance of A Resistor

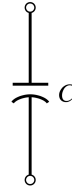
$$Z_R(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} = R\angle 0 = R$$



Capacitor ABC



(a) Basic construction



(b) Symbol

Capacitance C

A measure of how much charge a capacitor can hold.

▶ Amount of charge $Q = C \cdot V$

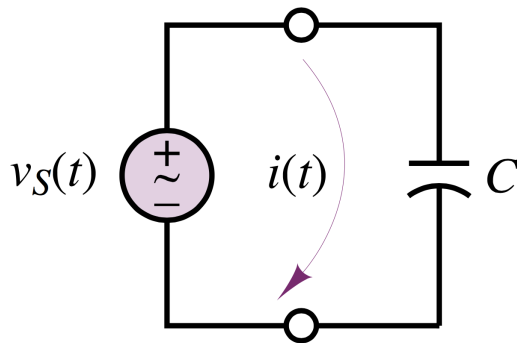
▶ **current** is the rate of movement of charge: $I = \frac{dQ}{dt} = C \cdot \frac{dV}{dt}$



Capacitor Impedance

$$\mathbf{V}(j\omega) = A\angle 0$$

$$\mathbf{I}(j\omega) = \omega CA\angle \pi/2$$



Impedance of A Capacitor

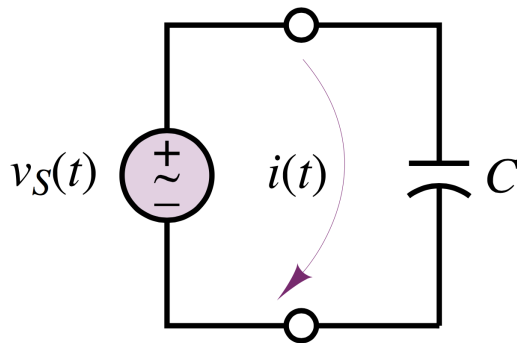
$$Z_C(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)}$$



Capacitor Impedance

$$\mathbf{V}(j\omega) = A\angle 0$$

$$\mathbf{I}(j\omega) = \omega CA\angle \pi/2$$

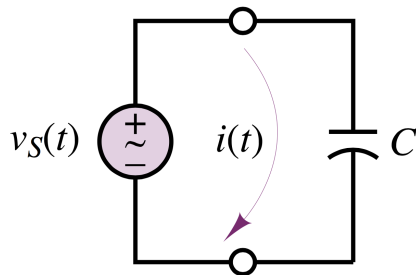


Impedance of A Capacitor

$$\begin{aligned} Z_C(j\omega) &= \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} \\ &= \frac{1}{\omega C} \angle -\pi/2 = \frac{1}{\omega C} [\cos(-\pi/2) + j\sin(-\pi/2)] \\ &= \frac{-j}{\omega C} = \frac{1}{j\omega C} \end{aligned}$$



$$Z_C(j\omega) = \frac{1}{j\omega C}$$



Capacitor Rule 1

Low Frequency \Rightarrow Open circuit

Capacitor Rule 2

High Frequency \Rightarrow Short circuit



Overview

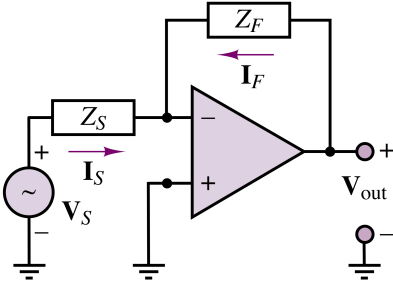
Preliminaries

Integrator & Differentiator

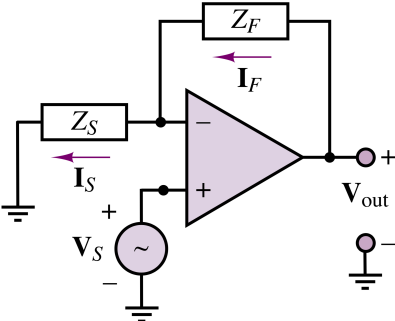
Filters



Frequency Response of An Op-Amp



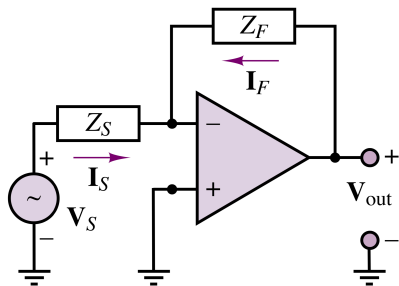
Inverting



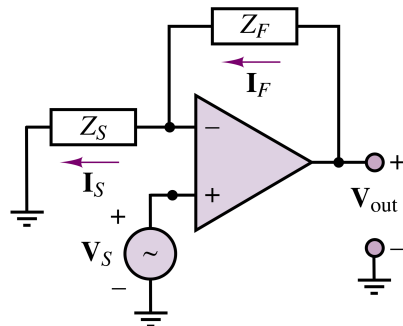
Noninverting



Frequency Response of An Op-Amp



Inverting

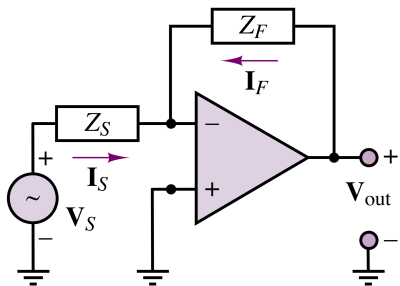


Noninverting

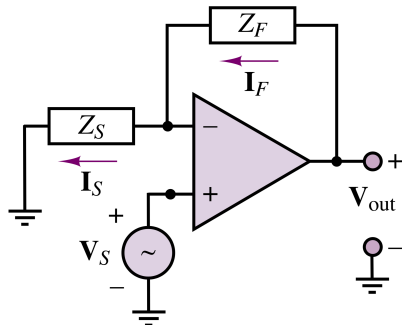
- ▶ Inverting amplifier:
$$\frac{V_{out}(j\omega)}{V_S} = -\frac{Z_F}{Z_S}$$



Frequency Response of An Op-Amp



Inverting

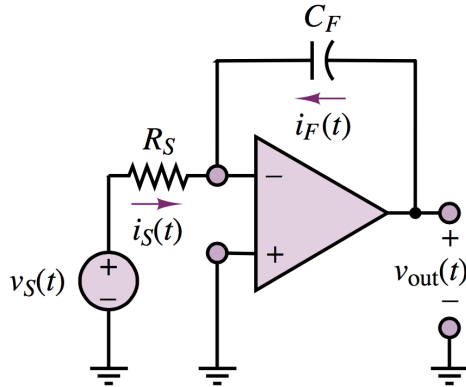


Noninverting

- ▶ Inverting amplifier: $\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$
- ▶ Non-Inverting amplifier: $\frac{V_{out}}{V_S}(j\omega) = 1 + \frac{Z_F}{Z_S}$



Integrator



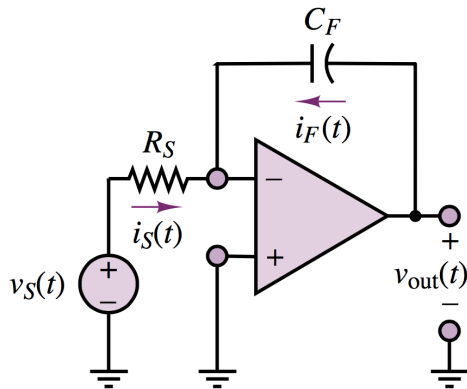
$$i_S(t) = -i_F(t)$$

$$i_S(t) = \frac{v_S(t)}{R_S}$$

$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$



Integrator



$$i_S(t) = -i_F(t)$$

$$i_S(t) = \frac{v_S(t)}{R_S}$$

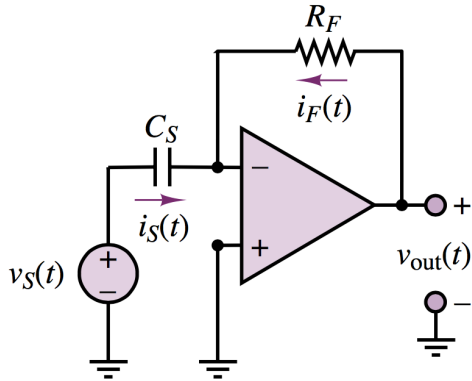
$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$

Therefore:

$$v_{out}(t) = -\frac{1}{R_S C_F} \int_{-\infty}^t v_S(t') dt'$$



Differentiator

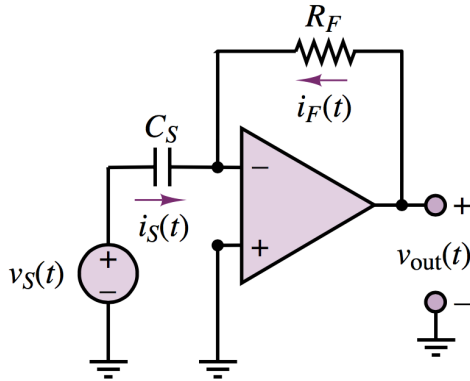


$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

$$i_F(t) = \frac{v_{out}(t)}{R_F}$$



Differentiator



$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

$$i_F(t) = \frac{v_{out}(t)}{R_F}$$

Therefore:

$$v_{out}(t) = -R_F C_S \cdot \frac{dv_S(t)}{dt}$$



Overview

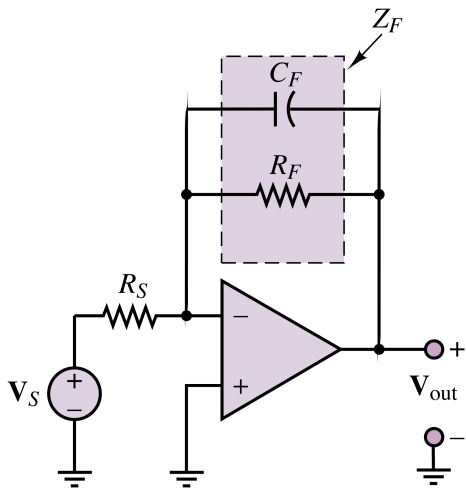
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Low-Pass Filter



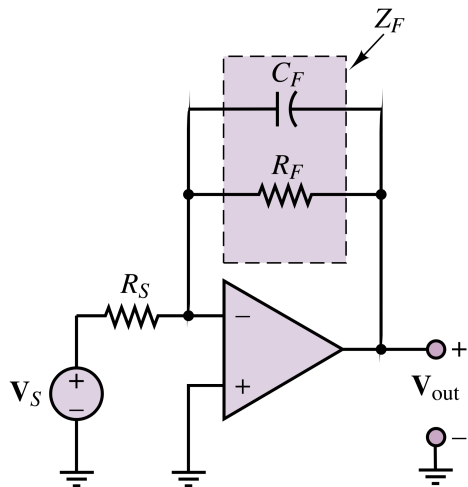
$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$



Low-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$



$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$



Given:

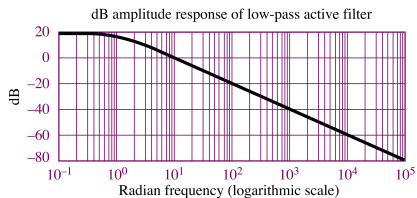
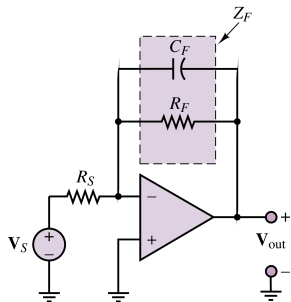
$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$
$$w_c = \frac{1}{R_F C_F}$$

Prove:

$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + \omega^2/w_c^2}}$$



Low-Pass Filter



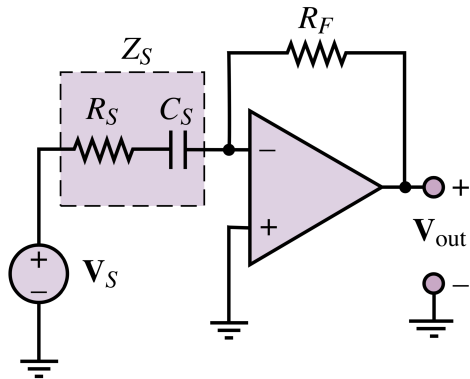
$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + \omega^2/w_c^2}}$$

- ▶ $w_c = \frac{1}{R_F C_F}$
- ▶ **3-dB** frequency
- ▶ or **cutoff** frequency

BTW, $\lim_{\omega \rightarrow 0} |A| = \frac{R_F}{R_S}$, $\lim_{\omega \rightarrow \infty} |A| = 0$



High-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_S = R_S + \frac{1}{j\omega C_S}$$

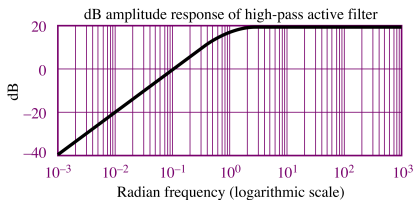
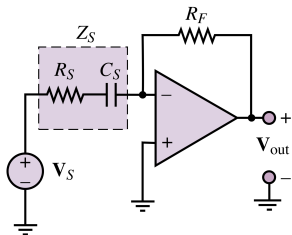
$$Z_F = R_F$$



$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$



High-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$

$$\lim_{\omega \rightarrow 0} |A| = 0$$

$$\lim_{\omega \rightarrow \infty} |A| = \frac{R_F}{R_S}$$

High freq. cutoff unintentionally created by Op-amp



Band-Pass Filter

