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Welfare of Price Discrimination and Market Segmentation in Duopoly

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Abstract

We study welfare consequences of third-degree price discrimination and market segmentation in a duopoly market with captive and contested consumers. A market segmentation divides the market into segments that contain different proportions of captive and contested consumers. Firm-optimal segmentation divides the market into two segments and in each segment only one firm has captive consumers. In contrast to the existing literature with exogenous segmentation, price discrimination under firm-optimal segmentation unambiguously reduces consumer surplus for all markets. Consumer-optimal segmentation divides the market into a maximal symmetric segment and the remainder, and yields the lowest producer surplus among all segmentations.

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1 Introduction

Third-degree price discrimination is ubiquitous and is probably the most common form of price discrimination. Almost all firms with some market power would attempt to increase profit by charging different prices for consumers in different sub-markets (or market segments). Hence, the welfare analysis of price discrimination is an important topic in the study of industrial organization.

Before the era of big data, consumers were segmented into different sub-markets by easily observable characteristics such as their ages and locations. With the advance of information technology and social media, the amount of consumer data available for firms to differentiate consumers grows exponentially, and the number of ways for firms to segment the market is enormous. Firms can identify consumers by consumers' digital footprints such as IP addresses, web browsing activities, social network and media posts, and combine them with offline consumer information to perform increasingly fine and intricate market segmentations. Hence, it is important to allow for flexible market segmentation in evaluating the welfare consequences of price discrimination.

Following the seminal work of Pigou (1920) and Robinson (1933), most of the literature on third-degree price discrimination takes the segmentation of consumers into different groups as exogenously given. The only issue for firms is to decide what prices to charge for each market segment. With exogenous market segmentation and general downward-sloping demand, the welfare consequences of price discrimination are ambiguous in both the monopoly setting,[1] and the oligopoly setting.[2]

The choice of how to divide the market, however, is clearly a very important consideration for firms (and data brokers) who can choose what kind of consumer data to collect, keep and process, and for regulators who can limit the nature and extent of consumer data to be collected, traded and used. Given the vastly many potential ways for firms or data brokers to segment the market, we will take an agnostic view and consider all possible segmentations in evaluating the welfare impact of price discrimination.

[1]The effect of monopolistic price discrimination on social welfare, relative to uniform pricing, depends on whether the overall output increases (Schmalensee (1981), Varian (1985)), or on the relative curvature of the direct or inverse demand functions in the two sub-markets (Aguirre, Cowan and Vickers (2010)). Consumer surplus must fall with price discrimination if discrimination lowers social welfare. But consumer surplus may rise with discrimination if the ratio of pass-through to the elasticity at the uniform price is higher in the high-elasticity sub-market (Cowan (2012)).

[2]In a symmetric duopoly model, Holmes (1989) shows that the effects of price discrimination on output and profit depend on cross-price elasticities and concavities of demand functions. Corts (1998) shows that if firms disagree over which sub-markets are strong or weak, then price discrimination may lower profit and increase consumer surplus. Armstrong and Vickers (2019) consider the baseline duopoly model we describe below and show that price discrimination hurts consumers if firms are sufficiently symmetric and benefits consumers if firms are sufficiently asymmetric.

In our baseline model, two firms produce a homogeneous product and compete in prices. Each firm has their own captive consumers who can only buy from the firm they are captive to. There are also contested consumers who are loyal to neither firms and will buy from the firm that offers the lower price. All consumers have the same downward-sloping demand.[3] Following Armstrong and Vickers (2019), we can equivalently view firms as competing in profit offers rather than in price offers.

A market segmentation divides the market into segments that contain different proportions of captive and contested consumers. We characterize the *unique* firm-optimal segmentation and the *unique* consumer-optimal segmentation among all possible segmentations. Both segmentations take simple forms. To succinctly describe them, let $(\gamma_1, 1-\gamma_1-\gamma_2, \gamma_2)$ denote a prior market where γ_i is the share of consumers captive to firm i and $1-\gamma_1-\gamma_2$ is the share of contested consumers. Let $\ell = \gamma_1+\gamma_2$ denote the total share of captive consumers. The firm-optimal segmentation divides the market into sub-market $(\ell, 1-\ell, 0)$ and sub-market $(0, 1-\ell, \ell)$ with size γ_1/ℓ and γ_2/ℓ, respectively.[4] In contrast, the consumer-optimal segmentation divides the market into the maximal symmetric sub-market and the remainder which is simply $(1,0,0)$ with size $(\gamma_1-\gamma_2)$ (if say $\gamma_1 > \gamma_2$).

The intuition is quite simple for the structure of the firm-optimal segmentation and for the reason why the consumer-optimal segmentation also generates the lowest producer surplus. In the unique mixed strategy equilibrium of any given segment, the dominant firm with more captive consumers always receives the minimum profit which it can secure by serving only its own captive consumers, while the profit for the dominated firm with fewer captive consumers is always higher than the profit by serving only its own captive consumers. As firms become more asymmetric, the dominant firm has weaker incentive to offer low profit to attract contested consumers, and hence competition becomes less intense and the total profit increases. Therefore, segments such as $(\ell, 1-\ell, 0)$ and $(0, 1-\ell, \ell)$ maximize such asymmetry (and hence profit) while symmetric segments minimize such asymmetry (and hence profit).

The above intuition, however, is insufficient to understand the effect of the firm-optimal segmentation on consumer surplus. As observed by Armstrong and Vickers (2019), consumer surplus, as a function of firms' profit, is generally concave. That

[3] This model framework is first developed by Narasimhan (1988) for the case of unit demand, and later generalized by Armstrong and Vickers (2019) to the case of downward-sloping demand. It has been a working horse in the marketing literature for studying promotional strategies (see for example, Chen, Narasimhan and Zhang (2001) and reference therein).

[4] This form of market segmentation is first noted by Armstrong and Vickers (2019). They observe that this segmentation arises if two regional monopolists are allowed to serve each other's customer bases, consumers differ in their switching costs, and firms engage in price discrimination by geographical regions.

is, consumers are risk averse with respect to variation in profit. A segmentation that yields a higher profit may not necessarily lead to a lower consumer surplus because it may also lower the variability of profit. Surprisingly, we are able to show that the firm-optimal segmentation always reduces consumer surplus compared to uniform pricing (i.e., no segmentation) for all prior markets, which stands in sharp contrast to the existing literature on price discrimination with exogenous sub-markets where the effect of price discrimination on consumer surplus is generally ambiguous.

The above intuition also hints at the critical role of the symmetric segment in maximizing consumer surplus, but it is only indicative. To characterize the unique consumer-optimal segmentation, we first show that a process of symmetrization by gradually taking out the dominant firm's captive consumers increases consumer welfare. We then argue that the symmetric segment must be maximal by drawing an insight of Armstrong and Vickers (2019) who observe that the profit distribution of several symmetric sub-markets is a mean-preserving spread of the profit distribution in the single symmetric market.

Methodologically, we follow the seminal work of Bergemann, Brooks and Morris (2015) (BBM hereafter) to formulate the segmentation problem as an information design problem. Instead of applying the standard concavification technique in the information design literature, we take a different approach.[5] We first identify the forms of market segments that can possibly be part of the optimal segmentation and then reformulate the information design problem as a problem of choosing the distributions of these segments. Our two-step solution procedure, more elementary and intuitive in our setup, can easily establish uniqueness as we solve the optimal segmentation. The uniqueness property is important for our welfare analysis, because, for example, different firm-optimal segmentations may have different profit distribution and hence different welfare implications for consumers.

In a monopoly setting with unit demand, BBM show that any surplus division (or equivalently any point in the surplus triangle) can be attained by some market segmentation. The analysis of BBM has been applied to a wide range of monopoly applications, such as multiproduct monopoly (Ichihashi (2020), Haghpanah and Siegel (2020), Hidir and Vellodi (2020)), lemons market with interdependent values (Kartik and Zhong (2019)), and revenue-maximizing data brokers (Yang (2020)).[6]

The extension of the analysis of BBM to the oligopoly setting, though natural and important, is technically challenging. As observed by Armstrong and Vickers (2019),

[5] See Bergemann and Morris (2019) and Kamenica (2019) for surveys of standard solution techniques and recent developments in this literature.

[6] See also Ali, Lewis and Vasserman (2020) for an analysis of how consumer information control can affect consumer welfare by influencing the learning of and the competition between firms.

even for duopoly pricing models, "[e]xcept in symmetric and other special cases ... the form of the equilibrium is not known." A stylized baseline model is hence necessary for tractability. Elliott and Galeotti (2019) consider a Hotelling model with captive consumers at the two ends and identify conditions for full surplus extraction through market segmentation. Albrecht (2020), and Bergemann, Brooks and Morris (2020*a*), Bergemann, Brooks and Morris (2020*b*) use the unit demand version of Armstrong and Vickers (2019) as their baseline model, and identify the firm- and consumer-optimal segmentations among all possible market segmentations based on public and private signals. Unit demand, however, is very special and has zero elasticity everywhere except at the jump. As demonstrated by the literature of price discrimination, elasticities and curvatures of demand are crucial in evaluating the welfare consequences of price discrimination. We allow for general downward-sloping demand but restrict attention to segmentations that are based on public signals.

All the above papers take consumer demand as given and study how to design information structures to influence learning by firms. One can also consider the design of information structures to affect consumer learning. Roesler and Szentes (2017) consider a monopoly model with privately informed consumers and derive consumer-optimal information structures. Armstrong and Zhou (2019) extend their analysis to a duopoly setting and characterize firm-optimal and consumer-optimal information structures. Assuming that firms rather than the designer choose information structures, Ivanov (2013) and Boleslavsky, Hwang and Kim (2019) derive equilibrium information structures in games where firms compete in both pricing and advertising.

2 The Model

Our baseline model is taken from Armstrong and Vickers (2019). There are two firms who can produce a homogeneous product at zero cost and compete for consumers in prices. There are three types of consumers: consumers who are captive to (and hence can only buy from) firm 1, consumers who are captive to firm 2, and contested consumers who will buy from the firm that charges a lower price. Let γ_1 and γ_2 denote the share of consumers captive to firm 1 and firm 2, respectively, and the share of contested consumers is then $1-\gamma_1-\gamma_2$. Without loss of generality, we assume that $\gamma_2 \leq \gamma_1$.

Consumers have quasilinear preferences and their demand $D(p)$ is downward sloping and continuously differentiable. If a consumer buys from a firm who charges price p, this consumer will buy $D(p)$ units of the product, yielding a profit of $\pi(p) \equiv pD(p)$ to the firm. As in Armstrong and Vickers (2019), we impose the following assumption:

Assumption 1 *The elasticity of demand $\eta(p) \equiv -pD'(p)/D(p)$ is strictly increasing.*

Under Assumption 1, $\pi(p)$ is single-peaked and hence is strictly increasing for all $p \in [0, p^*]$ where p^* is the revenue-maximizing price $p^* = \arg\max \pi(p)$. Moreover, consumer surplus $V(\pi)$ as a function of profit π is strictly concave in $[0, \pi^*]$, where $\pi^* \equiv p^* D(p^*)$ is the maximal profit.

The overall duopoly market, referred to as the prior market, can be segmented into different sub-markets or market segments which may have different relative shares of captive and relative consumers. We will use the terms of "sub-market" and "market segment" interchangeably. In a market segment $(q_1, 1-q_1-q_2, q_2)$, q_1 and q_2 are the fraction of consumers captive to firm 1 and firm 2, respectively, and $(1-q_1-q_2)$ is the fraction of contested consumers. To simplify notation, we write a market segment $(q_1, 1-q_1-q_2, q_2)$ as (q_1, q_2) and a prior market $(\gamma_1, 1-\gamma_1-\gamma_2, \gamma_2)$ as (γ_1, γ_2). The set of possible market segments is

$$\mathcal{M} = \left\{(q_1, q_2) \in [0,1]^2 : 0 \leq q_1 + q_2 \leq 1\right\}.$$

A market segmentation can be represented as a size distribution $m(q_1, q_2) \in \Delta\mathcal{M}$ of different segments such that, for $i = 1, 2$,

$$\gamma_i = \sum_{(q_1,q_2)\in\mathcal{M}} m(q_1, q_2)\, q_i.$$

For tractability, we assume that, once a market segmentation is chosen, it is publicly observable to both firms. This assumption is appropriate if information or signals on which the market segmentation is based are shared or publicly observable.

Given a market segmentation m, firms decide what prices to charge for each sub-market (q_1, q_2) in the support of m to maximize their profit. The producer surplus under segmentation m is

$$P(m) = \sum_{(q_1,q_2)\in\mathcal{M}} m(q_1, q_2)\left[\pi_1(q_1, q_2) + \pi_2(q_1, q_2)\right],$$

where $\pi_1(q_1, q_2)$ and $\pi_2(q_1, q_2)$ denote the profit in market segment (q_1, q_2) for firm 1 and firm 2, respectively. The total consumer surplus under segmentation m is

$$C(m) = \sum_{(q_1,q_2)\in\mathcal{M}} m(q_1, q_2)\, C(q_1, q_2)$$

where $C(q_1, q_2)$ denotes the consumer surplus in market segment (q_1, q_2).

A market segmentation is *firm-optimal* if it maximizes producer surplus among all possible market segmentations. A market segmentation is *consumer-optimal* if it maximizes consumer surplus among all possible market segmentations.

3 Firm- and Consumer-Optimal Segmentations

We first characterize the unique equilibrium for a generic market segment (q_1, q_2). The equilibrium characterization is then used to find the firm-optimal segmentation and the consumer-optimal segmentation.

3.1 Preliminaries

Fix a market segment (q_1, q_2) with $q_2 \leq q_1$. As demonstrated in Armstrong and Vickers (2019), it is more convenient to view firms as choosing the per-customer profit π rather than the price p they ask from their customers, and consumers choose the firm who offers the lowest profit among the firms they can buy from. The following equilibrium characterization is standard and is taken from Narasimhan (1988) and Armstrong and Vickers (2019). We omit its proof.

Lemma 1 *In the unique equilibrium for market segment (q_1, q_2) with $q_1 \geq q_2$, both firm 1 and firm 2 play mixed strategies on a common support $[\underline{\pi}, \pi^*]$ where the minimum profit $\underline{\pi} = q_1\pi^* / (1 - q_2)$. Firm 1 chooses per-consumer profit according to distribution*

$$F_1(\pi) = \frac{1 - q_1}{1 - q_1 - q_2}\left(1 - \frac{\underline{\pi}}{\pi}\right)$$

with an atom of size $(q_1 - q_2) / (1 - q_2)$ at $\pi = \pi^*$, and firm 2 chooses per-consumer profit according to distribution

$$F_2(\pi) = \frac{1 - q_2}{1 - q_1 - q_2}\left(1 - \frac{\underline{\pi}}{\pi}\right)$$

with no atom. The equilibrium profits are $\pi_1 = q_1\pi^*$ and $\pi_2 = (1 - q_1) q_1\pi^* / (1 - q_2)$.

It follows from Lemma 1 that the equilibrium producer surplus obtained in market segment (q_1, q_2) is

$$P(q_1, q_2) = \pi_1(q_1, q_2) + \pi_2(q_1, q_2) = \frac{(2 - q_1 - q_2) q_1}{1 - q_2}\pi^*. \tag{1}$$

Let $G(\pi; q_1, q_2)$ denote the equilibrium probability that a consumer in market segment (q_1, q_2) is offered a minimum profit weakly lower than π. Since firm i's profit offer is considered only by consumers captive to firm i and contested consumers, we have

$$\begin{aligned}
G(\pi;q_1,q_2) &= (1-q_2)F_1(\pi)+(1-q_1)F_2(\pi)-(1-q_1-q_2)F_1(\pi)F_2(\pi) \\
&= \frac{(1-q_1)(1-q_2)}{1-q_1-q_2}\left(1-\frac{q_1^2}{(1-q_2)^2}\left(\frac{\pi^*}{\pi}\right)^2\right) \qquad (2)
\end{aligned}$$

with an atom of size $q_1(q_1-q_2)/(1-q_2)$ at $\pi=\pi^*$. Therefore, the equilibrium consumer surplus is

$$\begin{aligned}
C(q_1,q_2) &= \int_{\underline{\pi}}^{\pi^*} V(\pi)\,dG(\pi;q_1,q_2) \\
&= \frac{q_1(q_1-q_2)}{1-q_2}V(\pi^*)+\frac{2q_1^2(1-q_1)}{(1-q_2)(1-q_1-q_2)}(\pi^*)^2\int_{\frac{q_1}{1-q_2}\pi^*}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi. \qquad (3)
\end{aligned}$$

We now introduce three forms of market segments that will play an important role in our characterization of firm- and consumer-optimal segmentations. We follow Armstrong and Vickers (2019) and call a market segment *nested* if the dominant firm's set of reachable consumers contains the dominated firms' set of reachable consumers. In other words, a market segment (q_1,q_2) is *nested* if either $q_1=0$ or $q_2=0$. A market segment (q_1,q_2) is *symmetric* if $q_1=q_2$. A market segment (q_1,q_2) is *perfect* if it contains only one type of consumers (i.e., either $q_1=1$, or $q_2=1$, or $q_1+q_2=1$).

The following lemma identifies some important properties of the nested segment and the symmetric segment. It will be repeatedly used for our later characterization.

Lemma 2 *Producer surplus is strictly concave in q for a nested segment, $(q,0)$ or $(0,q)$. Producer surplus is linear in q and consumer surplus is strictly concave in q for a symmetric segment (q,q).*

Proof. For a nested segment, producer surplus $P(q,0)=(2-q)q\pi^*$, which is strictly concave in q. For a symmetric segment, producer surplus is $2q\pi^*$, which is linear in q. Consumer surplus is given by

$$C(q,q)=\frac{2q^2}{1-2q}(\pi^*)^2\int_{\frac{q}{1-q}\pi^*}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi.$$

To show it is strictly concave in q, we need to show that, for any $\lambda\in[0,1]$, any $0<q_L<q_H<1/2$ and $q=\lambda q_L+(1-\lambda)q_H$,

$$C(q,q)>\lambda C(q_L,q_L)+(1-\lambda)C(q_H,q_H).$$

In other words, if we segment a symmetric prior market (q,q) into two symmetric sub-markets, (q_L, q_L) with size λ and (q_H, q_H) with size $(1-\lambda)$, then consumer surplus must decrease. Since $V(\pi)$ is concave, it is sufficient to show that the minimum profit distribution in the two sub-markets is a strict mean-preserving spread of the minimum profit distribution in the single prior market.

Let $G(\pi;q) \equiv G(\pi;q,q)$ denote the probability that a consumer in market segment (q,q) is offered a minimum profit weakly lower than π. Then we must have

$$G(\pi;q) = \frac{(1-q)^2}{1-2q} - \frac{q^2}{1-2q}\left(\frac{\pi^*}{\pi}\right)^2.$$

$G(\pi;q)$ is strictly concave in q for all $\pi < \pi^*$ because

$$\frac{\partial^2 G(\pi;q)}{\partial q^2} = -\left(\frac{2}{1-2q} + \frac{8q(1-q)}{(1-2q)^3}\right)\left(\left(\frac{\pi^*}{\pi}\right)^2 - 1\right) < 0.$$

Consider $q_L < q_H$, $\lambda \in (0,1)$ and $q = \lambda q_L + (1-\lambda) q_H$. Then for all $\pi \in \left[\frac{q_H}{1-q_H}\pi^*, \pi^*\right]$,

$$\overline{G}(\pi;q) \equiv \lambda G(\pi;q_L) + (1-\lambda) G(\pi;q_H) < G(\pi;q).$$

Since $q > q_L$, the support of $\overline{G}(\pi;q)$ contains the support of $G(\pi;q)$. Furthermore, for $\pi \in \left[\frac{q}{1-q}\pi^*, \frac{q_H}{1-q_H}\pi^*\right]$,

$$\frac{G'(\pi;q)}{\overline{G}'(\pi;q)} = \frac{1}{\lambda}\frac{q^2}{1-2q}\left(\frac{q_L^2}{1-2q_L}\right)^{-1} > 1$$

because function $f(x) = x^2/(1-2x)$ is strictly increasing and $q > q_L$. It follows that $G(\pi;q)$ crosses $\overline{G}(\pi;q)$ only once and from below. Finally, the two sub-markets yield the same producer surplus of $2q\pi^*$ as the prior single symmetric market (q,q). Therefore, $\overline{G}(\pi;q)$ is a mean-preserving spread of $G(\pi;q)$. The strict concavity of $V(\pi)$ then implies that

$$C(q,q) > \lambda C(q_L,q_L) + (1-\lambda) C(q_H,q_H).$$

That is, $C(q,q)$ is strictly concave in q. ■

The argument for the concavity of $C(q,q)$ is first sketched out in Armstrong and Vickers (2019) who observe that the distribution of offered profit across several symmetric sub-markets is a mean-preserving spread of the profit distribution in the single symmetric market.

The strict concavity of producer surplus for nested segments implies that a merger of two different nested segments always strictly increases producer surplus. The strict concavity of consumer surplus and linearity of producer surplus for symmetric segments imply that a merger of two different symmetric segments always strictly increases consumer surplus, while leaving producer surplus unchanged.

It is easy to see that if a prior market (γ_1, γ_2) does not contain any contested consumers (i.e., $\gamma_1 + \gamma_2 = 1$), both firms will offer the maximal profit π^* for every market segment. Therefore, all market segmentations yield the same payoffs for firms and consumers. From now on, we assume that $\gamma_1 + \gamma_2 < 1$.

3.2 Firm-Optimal Segmentation

Fix a prior market (γ_1, γ_2) with $\gamma_1 \geq \gamma_2$. To characterize the (unique) firm-optimal segmentation, we proceed in two steps. First, we argue that every market segment that is part of a firm-optimal segmentation must be a nested segment. Second, we reformulate the designer's problem and show that a firm-optimal segmentation either contains one nested segment (if the prior market is nested) or two nested segments which take the form of $(\gamma_1 + \gamma_2, 0)$ and $(0, \gamma_1 + \gamma_2)$.

Lemma 3 *Every sub-market in a firm-optimal segmentation is a nested segment.*

Proof. We first show that a sub-market (q_1, q_2) with $q_1 > 0$, $q_2 > 0$ and $q_1 + q_2 < 1$ cannot be part of a firm-optimal segmentation. If it is, we can further decompose it into two sub-markets as follows:

$$(q_1, q_2) = \frac{q_1}{q_1 + q_2} (q_1 + q_2, 0) + \frac{q_2}{q_1 + q_2} (0, q_1 + q_2) .$$

The decomposition yields a strictly higher producer surplus because

$$\begin{aligned} & \frac{q_1}{q_1 + q_2} P (q_1 + q_2, 0) + \frac{q_1}{q_1 + q_2} P (0, q_1 + q_2) - P (q_1, q_2) \\ = \;& \frac{q_2}{1 - q_2} (1 - q_1 - q_2) (2 - q_1 - q_2) \pi^* > 0. \end{aligned}$$

A contradiction to the optimality. It remains to show that a sub-market $(q, 1 - q)$ with $q \in (0, 1)$ cannot be part of a firm-optimal segmentation. Suppose by contradiction that a firm-optimal segmentation includes such a sub-market. We can decompose this sub-market into $q (1, 0) + (1 - q) (0, 1)$ and maintain the same producer surplus. Since $\gamma_1 + \gamma_2 < 1$, a firm-optimal segmentation must involve at least one nested segment with strictly positive amount of shoppers. Without loss of generality, suppose this nested segment takes the form of $(x, 1-x, 0)$ with $x < 1$. Since both segments $(1, 0, 0)$ and $(x, 1-x, 0)$ are nested segments, by Lemma 2, a merger of the two yields a strictly higher producer surplus. Again a contradiction to the optimality. ■

To get intuition, consider all market segments (q_1, q_2) with $q_1 \geq q_2$ that have the same total share of captive consumers, that is, $q_1 + q_2 = \ell$ for some constant $\ell \in (0, 1)$. Then the larger q_1 is, the more asymmetric the segment becomes. As q_1 increases (keeping ℓ fixed), the incentive of firm 1 to attract contested consumers with low profit offers decreases, the equilibrium profit distribution shifts up. Formally, by (1), the total profit of segment (q_1, q_2) is

$$P(q_1, q_2) = \frac{(2-\ell)\, q_1}{1-\ell+q_1}\pi^*,$$

which is increasing in q_1. Nested segments such as $(\ell, 0)$ and $(0, \ell)$ has the maximal asymmetry and hence maximize profit while symmetric segments minimizes profit.

Given Lemma 3, it is natural to consider the following segmentation for a prior market (γ_1, γ_2):

$$(\gamma_1, \gamma_2) = \frac{\gamma_1}{\gamma_1+\gamma_2}(\gamma_1+\gamma_2, 0) + \frac{\gamma_2}{\gamma_1+\gamma_2}(0, \gamma_1+\gamma_2).$$

Since all sub-markets are nested segments, the above segmentation will be referred to as a "*nested segmentation.*"

Proposition 1 *The nested segmentation uniquely maximizes producer surplus among all possible market segmentations.*

Proof. Lemmas 2 and 3 imply that a firm-optimal segmentation can contain at most two nested segments and if there are two nested segments they must take the form of $(q, 0)$ and $(0, q')$ with $q, q' \in [0, 1]$. Therefore, the designer's problem of finding all firm-optimal segmentations can be simplified as

$$\max_{(q,m;q',m')} mP(q, 0) + m'P(0, q')$$

subject to $\gamma_1 = mq$, $\gamma_2 = m'q'$ and $m + m' = 1$. By substituting the three constraints and expressions for $P(q, 0)$ and $P(0, q')$, we can rewrite the objective as a function of q as

$$\gamma_1(2-q)\pi^* + \gamma_2\left(2 - \frac{\gamma_2}{1-\gamma_1/q}\right)\pi^*$$

which is strictly concave for all $q \geq \gamma_1$ and has a unique maximizer of $q = \gamma_1 + \gamma_2$. Therefore, the unique firm-optimal segmentation must take the form of a nested segmentation. ■

We conclude this subsection by showing that the firm-optimal segmentation always makes consumers worse off relative to uniform pricing without segmentation.

Proposition 2 *The unique firm-optimal segmentation yields a lower consumer surplus than uniform pricing for any prior market.*

Proof. Note that consumer surplus under the first-optimal segmentation is $C(\gamma_1+\gamma_2,0)$, so we need to show $C(\gamma_1,\gamma_2)\geq C(\gamma_1+\gamma_2,0)$. Define $\ell=\gamma_1+\gamma_2$ and rewrite

$$C(\gamma_1,\gamma_2)=C(\gamma_1,\ell-\gamma_1).$$

We prove that $C(\gamma_1,\ell-\gamma_1)$ is decreasing in γ_1 for fixed ℓ. That is, for a fixed total share of captive consumers, consumer surplus decreases as the distribution of captive consumers becomes more uneven between the two firms. We can use (3) to write

$$C(\gamma_1,\ell-\gamma_1)=\frac{\gamma_1(2\gamma_1-\ell)}{1-\ell+\gamma_1}V(\pi^*)+\frac{2\gamma_1^2(1-\gamma_1)(\pi^*)^2}{(1-\ell)(1-\ell+\gamma_1)}\int_{\underline{\pi}}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi,$$

where the minimum profit $\underline{\pi}$ is a function of γ_1 and ℓ:

$$\underline{\pi}=\frac{\gamma_1}{1-\ell+\gamma_1}\pi^*.$$

We take the total derivative with respect to γ_1:

$$\begin{aligned}\frac{dC(\gamma_1,\ell-\gamma_1)}{d\gamma_1} &= \frac{2\gamma_1^2+(4\gamma_1-\ell)(1-\ell)}{(1-\ell+\gamma_1)^2}V(\pi^*)-\frac{2(1-\gamma_1)}{\gamma_1}V(\underline{\pi})\\ &\quad+\frac{2(2\gamma_1-3\gamma_1^2)(1-\ell+\gamma_1)-2\gamma_1^2(1-\gamma_1)}{(1-\ell+\gamma_1)^2(1-\ell)}(\pi^*)^2\int_{\underline{\pi}}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi.\end{aligned}$$

Since $V(\pi)$ is decreasing in π, we have $V(\pi)\leq V(\underline{\pi})$ for all $\pi\in[\underline{\pi},\pi^*]$. It follows that

$$(\pi^*)^2\int_{\underline{\pi}}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi\leq V(\underline{\pi})(\pi^*)^2\int_{\underline{\pi}}^{\pi^*}\frac{1}{\pi^3}d\pi=V(\underline{\pi})\frac{(1-\ell)(2\gamma_1-\ell+1)}{2\gamma_1^2}.$$

Noting that the coefficient for the integral is positive and $V(\pi^*)\leq V(\underline{\pi})$, we obtain that

$$\frac{dC(\gamma_1,\ell-\gamma_1)}{d\gamma_1}\leq 0\cdot V(\underline{\pi})=0.$$

Therefore, $C(\gamma_1,\gamma_2)\geq C(\gamma_1+\gamma_2,0)$. ■

The intuition for $C(\gamma_1,\gamma_2)\geq C(\gamma_1+\gamma_2,0)$ is as follows. Consider a market (γ_1,γ_2) with $\gamma_1>\gamma_2$. Suppose we increase γ_1 but keep $\ell=\gamma_1+\gamma_2$ unchanged. There are two effects associated with an increase in γ_1. First, as we argue earlier, when the market becomes more asymmetric, the equilibrium profit distribution shifts up, which tends to lower consumer surplus. Second, as γ_1 increases, the support of profit distribution shrinks and the variability of profit may go down, which tends to benefit consumers who are risk averse in offered profit π. It turns out the first effect always dominates.

If for a nested segment $(\ell, 0)$ consumer surplus $C(\ell, 0)$ is convex in ℓ, then any further division of segment $(\ell, 0)$ can only increase consumer surplus. In this case, the firm-optimal segmentation also minimizes consumer surplus. But $C(\ell, 0)$ is not necessarily convex in ℓ. To see this, note that

$$C(\ell, 0) = \ell^2 V(\pi^*) + 2\ell^2 (\pi^*)^2 \int_{\ell\pi^*}^{\pi^*} \frac{V(\pi)}{\pi^3} d\pi,$$

and

$$\frac{\partial^2 C(\ell, 0)}{\partial \ell^2} = 2V(\pi^*) + 4(\pi^*)^2 \int_{\ell\pi^*}^{\pi^*} \frac{V(\pi)}{\pi^3} d\pi - \frac{2}{\ell^2} V(\ell\pi^*) - \frac{2\pi^*}{\ell} V'(\ell\pi^*).$$

It is easy to see that $C(\ell, 0)$ is convex in ℓ as $\ell \to 1$ and is concave as $\ell \to 0$. Therefore, if the share of captive consumers is large ($\ell \to 1$), then the nested segmentation is also consumer-surplus minimizing. If the share of captive consumers is small ($\ell \to 0$), however, there are alternative segmentations that generate lower consumer surplus than the nested segmentation.

3.3 Consumer-Optimal Segmentation

As for the firm-optimal segmentation, we follow a two-step procedure to find the unique consumer-optimal segmentation. We first show that every segment in a consumer-optimal segmentation must be either symmetric or perfect, and then we use this observation to simplify and solve the information design problem.

Armstrong and Vickers (2019) show that, if the prior market is symmetric, uniform pricing generates a higher consumer surplus than price discrimination for any market segmentation. This insight is partially captured in Lemma 2 and strongly suggests that a consumer-optimal segmentation must involve symmetric market segments.

Lemma 4 *Every sub-market in a consumer-optimal segmentation must be either symmetric or perfect.*

Proof. Suppose by contradiction that a consumer-optimal segmentation contains a segment that is neither symmetric nor perfect. Let (q_1, q_2) denote this segment. Then we must have $q_1, q_2 \in [0,1)$ and $q_1 \neq q_2$.Without loss we assume $q_1 > q_2$. Consider the following ε-segmentation:

$$(1-\varepsilon)\left(\frac{q_1-\varepsilon}{1-\varepsilon},, \frac{q_2}{1-\varepsilon}\right)+\varepsilon\,(1,0)$$

where $\varepsilon \in (0, q_1-q_2]$. Consumer surplus under the ε-segmentation is

$$\begin{aligned}
& C^{\varepsilon}(q_1,q_2) \\
= \;& (1-\varepsilon)\, C\left(\frac{q_1-\varepsilon}{1-\varepsilon}, \frac{q_2}{1-\varepsilon}\right)+\varepsilon V(\pi^*) \\
= \;& \frac{q_1-\varepsilon}{1-\varepsilon-q_2}\left((q_1-\varepsilon-q_2)\, V(\pi^*)+\frac{2\,(q_1-\varepsilon)\,(1-q_1)\,(\pi^*)^2}{1-q_1-q_2}\int_{\frac{q_1-\varepsilon}{1-\varepsilon-q_2}\pi^*}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi\right)+\varepsilon V(\pi^*)
\end{aligned}$$

With some algebra, we can show that

$$\begin{aligned}
\frac{\partial C^{\varepsilon}(q_1,q_2)}{\partial \varepsilon} = \;& \frac{(1-q_1-q_2)\,(1-q_1)}{(1-\varepsilon-q_2)^2}(q_1-\varepsilon-q_2)\, V(\pi^*)+\frac{2\,(1-q_1)}{(q_1-\varepsilon)}V\left(\frac{q_1-\varepsilon}{1-\varepsilon-q_2}\pi^*\right) \\
& -\frac{2\,(q_1-\varepsilon)\,(1-q_1)\,(2-q_1-2q_2-\varepsilon)}{(1-\varepsilon-q_2)^2\,(1-q_1-q_2)}(\pi^*)^2\int_{\frac{q_1-\varepsilon}{1-\varepsilon-q_2}\pi^*}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \frac{\partial C^{\varepsilon}(q_1,q_2)}{\partial \varepsilon}\Big|_{\varepsilon=0} \\
= \;& \frac{(1-q_1-q_2)\,(1-q_1)}{(1-q_2)^2}(q_1-q_2)\, V(\pi^*)+\frac{2\,(1-q_1)}{q_1}V\left(\frac{q_1}{1-q_2}\pi^*\right) \\
& -\frac{2q_1\,(1-q_1)\,(2-q_1-2q_2)}{(1-q_2)^2\,(1-q_1-q_2)}(\pi^*)^2\int_{\frac{q_1}{1-q_2}\pi^*}^{\pi^*}\frac{V(\pi)}{\pi^3}d\pi \\
\geq \;& \frac{2\,(1-q_1)}{q_1}V\left(\frac{q_1}{1-q_2}\pi^*\right)-\frac{2q_1\,(1-q_1)\,(2-q_1-2q_2)}{(1-q_2)^2\,(1-q_1-q_2)}V\left(\frac{q_1}{1-q_2}\pi^*\right)\int_{\frac{q_1}{1-q_2}\pi^*}^{\pi^*}\frac{(\pi^*)^2}{\pi^3}d\pi
\end{aligned}$$

where the inequality follows because the term involving $V(\pi^*)$ is non-negative and $V\left(\frac{q_1}{1-q_2}\pi^*\right) \geq V(\pi)$ for all $\pi \in \left[\frac{q_1}{1-q_2}\pi^*, \pi^*\right]$. Note that

$$\int_{\frac{q_1}{1-q_2}\pi^*}^{\pi^*}\frac{(\pi^*)^2}{\pi^3}d\pi=\frac{1-q_1^2+q_2^2-2q_2}{2q_1^2}.$$

Hence, after collecting coefficients for $V\left(\frac{q_1}{1-q_2}\pi^*\right)$, we deduce that

$$\frac{\partial C^{\varepsilon}\left(q_1,q_2\right)}{\partial\varepsilon}|_{\varepsilon=0}\geq\frac{\left(1-q_1\right)\left(1-q_1-q_2\right)}{\left(1-q_2\right)^2}V\left(\frac{q_1}{1-q_2}\pi^*\right)>0.$$

Therefore, the ε-segmentation strictly increases consumer surplus, a contradiction to the optimality. ■

Now consider the following market segmentation for a prior market (γ_1,γ_2):

$$(\gamma_1,\gamma_2)=(1-\gamma_1+\gamma_2)\left(\frac{\gamma_2}{1-\gamma_1+\gamma_2},\frac{\gamma_2}{1-\gamma_1+\gamma_2}\right)+(\gamma_1-\gamma_2)\,(1,0)\,,$$

which contains a symmetric segment and a perfect segment. Since the symmetric segment levels the playing field for the two firms, we will refer to the above segmentation as a "*field-levelling segmentation.*"

Proposition 3 *The field-levelling segmentation uniquely maximizes consumer surplus among all possible market segmentations.*

Proof. By Lemma 2, a consumer-optimal segmentation can contains only one symmetric segment. Next, we argue that it cannot contain two perfect segments, $(0,1)$ and $(1,0)$, simultaneously. To see this, suppose by contradiction, it contains $(0,1)$ and $(1,0)$ with size x and y, respectively. Suppose that $x\geq y$ (the case of $x<y$ is analogous). We can replace $x(1,0)+y(0,1)$ with $(x-y)(1,0)+2y(\frac{1}{2},\frac{1}{2})$ and merge $(\frac{1}{2},\frac{1}{2})$ and the symmetric segment in the consumer-optimal segmentation to strictly increase consumer surplus according to Lemma 2 since they are two different symmetric segments. Finally, since $\gamma_1\geq\gamma_2$ by assumption, the consumer-optimal segmentation can contain only the perfect segment of the form $(1,0)$. Hence, the consumer-optimal segmentation must be a field-levelling segmentation. ■

If the segmentation is chosen optimally for the consumers, what would be its payoff implication for firms and the society overall? The following proposition answers the question.

Proposition 4 *The field-levelling segmentation minimizes producer surplus. If $V\left(\pi^*\right)\geq\pi^*$, the field-levelling segmentation also maximizes social surplus.*

Proof. We first use the ε-segmentation to show that, if $V\left(\pi^*\right)\geq\pi^*$, every segment in a social welfare maximizing segmentation must be either symmetric or perfect. Suppose not and consider any segment (q_1,q_2) with $q_1,q_2\in[0,1)$ and $q_1\neq q_2$. The

ε-segmentation

$$(1-\varepsilon)\left(\frac{q_1-\varepsilon}{1-\varepsilon},\frac{q_2}{1-\varepsilon}\right)+\varepsilon\,(1,0)$$

yields a producer surplus of

$$P^{\varepsilon}(q_1,q_2)=\pi^*\frac{q_1^2+q_1q_2-2q_1-\varepsilon}{1-q_2-\varepsilon}$$

with

$$\frac{\partial P^{\varepsilon}(q_1,q_2)}{\partial\varepsilon}|_{\varepsilon=0}=-\left(1-q_1\right)\frac{1-q_1-q_2}{\left(1-q_2\right)^2}\pi^*.$$

It follows that

$$\begin{aligned}
\frac{\partial S^{\varepsilon}(q_1,q_2)}{\partial\varepsilon}|_{\varepsilon=0} &= \frac{\partial P^{\varepsilon}(q_1,q_2)}{\partial\varepsilon}|_{\varepsilon=0}+\frac{\partial C^{\varepsilon}(q_1,q_2)}{\partial\varepsilon}|_{\varepsilon=0}\\
&\geq -\left(1-q_1\right)\frac{1-q_1-q_2}{\left(1-q_2\right)^2}\pi^*+\frac{\left(1-q_1\right)\left(1-q_1-q_2\right)}{\left(1-q_2\right)^2}V\left(\frac{q_1}{1-q_2}\pi^*\right)\\
&= \frac{\left(1-q_1\right)\left(1-q_1-q_2\right)}{\left(1-q_2\right)^2}\left(V\left(\frac{q_1}{1-q_2}\pi^*\right)-\pi^*\right)\\
&> 0.
\end{aligned}$$

The first inequality follows from the bound for $\frac{\partial C^{\varepsilon}(q_1,q_2)}{\partial\varepsilon}|_{\varepsilon=0}$ obtained in the proof of Lemma 4, and the last inequality follows from the fact that $V\left(\frac{\gamma_1}{1-\gamma_2}\pi^*\right)>V\left(\pi^*\right)\geq\pi^*$. By Lemma 2, producer surplus is linear in q and consumer surplus is strictly concave in q for a symmetric segment (q,q). Hence, social surplus $S\,(q,q)$ is also strictly concave in q. An argument similar to the one in the proof of Proposition 3 can be used to establish that the field-levelling segmentation maximizes social surplus. ■

Intuitively, when $V\left(\pi^*\right)\geq\pi^*$, consumer surplus carries a sufficiently large weight in the composition of social surplus. The gains of consumers from the process of gradually levelling the playing field outweigh the losses of firms. Since consumer surplus is strictly concave and producer surplus is linear for symmetric segment, social surplus is also strictly concave. As a result, the symmetric segment in the socially optimal segmentation must be maximal.

4 Concluding Remarks

This paper has characterized maximal producer surplus and maximal consumer surplus attainable by all possible market segmentations in a specialized duopoly model. We have also showed that the firm-optimal segmentation yields a lower consumer surplus than no segmentation, and that the consumer-optimal segmentation also minimizes producer surplus among all segmentations. A natural next step is to characterize the set of payoff vectors that are attainable by all market segmentations, as BBM do in their monopoly setting. With general downward-sloping demand, however, the task of characterizing the full payoff set is difficult.

If consumers in our model have unit demand as in BBM, then the set of attainable payoffs takes a simple form. With offered profits replaced by offered prices, the unique equilibrium for every market segment is the same as the one characterized in Lemma 1. All consumers are served in any market segment, so full surplus is always realized. This fact, together with risk-neutrality associated with unit demand, implies that the interests of firms and consumers are diametrically opposed. It follows that the nested segmentation attains the point of maximal producer surplus and minimal consumer surplus and that the field-levelling segmentation attains the point of minimal producer surplus and maximal consumer surplus. The set of attainable payoffs is then the line segment connecting these two points.

Throughout of the paper, we restrict attention to market segmentations that are based on signals observable to both firms. In many applications, firms often do not share the same information about consumers. As demonstrated by Bergemann, Brooks and Morris (2020*b*) in the unit demand setting, market segmentation based on private signals may strictly improve firms' profits. How would our welfare analysis of price discrimination change if we allow for private segmentation in the downward-sloping demand setting? This is an important research question and we leave it for future research.

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