Each question is worth 10 points. Please explain your solution clearly and concisely.

1. Show that $\sqrt{2} + \sqrt{6}$ is an irrational number.

2. You have an 8 litre jug and a 18 litre jug. The 18 litre jug has a leak: Whenever you pour ℓ litres out of it, ℓ additional litres disappear from it. Can you measure 1 litre using the rules from Lecture 4?

3. Show that in every graph, the sum of the squares of the degrees of all the vertices is an even number.

4. The vertices of graph H are the $2n$ integers from $-n$ to n except 0. The edges of H are the pairs $\{x, y\}$ such that $x = -y$ or $|y - x| = 1$. How many perfect matchings does H have?

5. Show that for every n, $1 + 1/2^3 + 1/3^3 + \cdots + 1/n^3 < 1.25$.

6. How many five-card poker hands are there that have the same number of kings and aces?

7. Let p be a polynomial of the form $p(x) = ax^4 + bx^3 + x^2$ over $\mathbb{F}_q$, where q is a prime number. Show that p has at most three zeros.

8. Alice comes up with a circular seating arrangement for n guests at a round table. Show that Bob needs to ask Alice $\Omega(n \log n)$ yes/no questions in order to figure out the arrangement with certainty.