ENGG 2440A: Discrete Mathematics for Engineers — Homework 1
The Chinese University of Hong Kong, Fall 2015 — due Thursday 24 September

Each of the questions is worth 10 points. Please turn in solutions to *four* questions of your choice. Write your name, your student ID, and your TA's name on the solution sheet.

Please write your solutions clearly and concisely. If you do not explain your answer you will be given no credit. You are encouraged to collaborate on the homework, but you must write your own solutions and list your collaborators on your solution sheet. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Questions

1. Say which among the following pairs of propositions are logically equivalent. Explain and justify your answer.

   (a) $(P \text{ OR } Q) \longrightarrow (P \text{ AND } Q)$
   $P \text{ XOR } Q$

   (b) $P \longrightarrow (\text{NOT } Q)$
   $Q \longrightarrow (\text{NOT } P)$

   (c) If Alice wants cake then Bob wants cake. If Bob wants cake then Charlie wants cake.
   If Alice wants cake then Charlie wants cake.

   (d) All balls are red or blue.
   All balls are red or all balls are blue.

2. Express the following propositions about people and their relative heights using quantifiers and logical operators. Use x, y, z as variables and $T(x, y)$ for "x is taller than y". Make sure that all your variables are quantified. Explain your answer.

   (a) No one is both taller and shorter than Bob.

   (b) Bob is not the tallest person and he is not the shortest person.

   (c) One cannot be both taller and shorter than someone.

   (d) There is a shortest person, but there is no tallest person.

   (e) There are at least two people that are taller than Bob.
   (You can use $x \neq y$ for "x and y are different people".)

   (f) (**Extra credit)** There are at most two people that are taller than Bob.

3. The following propositions are about employees and the cars they sell: $S(e, m)$ means "Employee e sold a car of make m" (for example $S(\text{Bob}, \text{Toyota})$ means "Bob sold a Toyota") and $P(e)$ means "Employee e was promoted".

   (a) Explain the meaning of these two propositions in plain English:
   $A\colon \forall m\ \exists e\colon S(e, m) \text{ AND } (\text{NOT } P(e))$
   $B\colon \exists m\ \forall e\colon S(e, m) \longrightarrow P(e)$

(b) Can both A and B be true? Justify your answer.

(c) Explain the meaning of these two propositions in plain English:
$C\colon \forall m\ \exists e\ \forall n\colon S(e,m)$ AND $((m \neq n) \longrightarrow (\text{NOT } S(e,n)))$
$D\colon \forall m,n\ \exists e\colon S(e,m)$ AND $((m \neq n) \longrightarrow (\text{NOT } S(e,n)))$

(d) Are C and D logically equivalent? Justify your answer.

4. Say which among the following pairs of propositions with quantifiers are logically equivalent. Explain and justify your answer.

   To argue that two propositions are equivalent, describe their common meaning in English. To argue that two propositions are not equivalent, describe a possible world in which one of them is true and the other one is false.

   (a) $\exists x\ \forall y\colon P(x,y)$
   $\exists y\ \forall x\colon P(y,x)$

   (b) $(\exists x\colon S(x))$ AND $(\exists x\colon T(x))$
   $\exists x\colon S(x)$ AND $T(x)$.

   (c) $(\exists x\colon S(x))$ AND $(\exists x\colon T(x))$
   $\exists x,y\colon S(x)$ AND $T(y)$.

   (d) $\forall x\ \exists y\colon S(x)$ AND $T(y)$
   $(\forall x\colon S(x))$ AND $(\exists y\colon T(y))$.

   (e) $\forall x\ \exists y\colon S(x)$ AND $T(y)$
   $\forall y\ \exists x\colon S(x)$ AND $T(y)$.

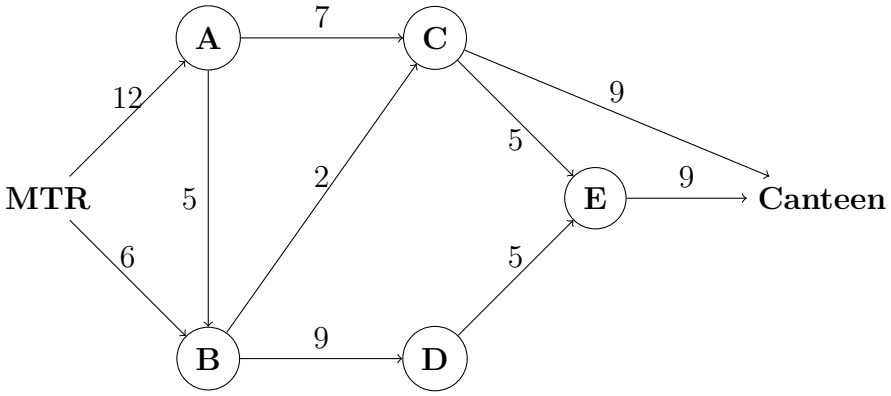
5. Alex, Bob and Chris, three students of logic, encounter the following situations:

   (a) Alex, Bob, and Chris stand in line with Alex in front, Bob in the middle, and Chris in the back. There are two white hats and two black hats. Each of Alex, Bob, and Chris has a hat on his head. Each one of them can only see the hats of the people ahead of him.

   Alex, Bob, and Chris are asked if they know the colour of their hat. After a while, Bob says "My hat is white." What is the colour of Alex's hat?

   (b) Alex, Bob, and Chris travel to an island where everyone is known to either always lie or always tell the truth. They encounter three inhabitants: Peter, Rachel, and Tom. Peter says "All three of us are liars." Then Tom says "At least one among us three tells the truth". Who is a liar and who tells the truth?

6. You are organising the distribution of watermelons around CUHK. You will hire a porter to get as many as possible from the University MTR station to the Medical School Canteen. Here is the network of trails that can be used to carry the watermelons:



The number above each arrow is the largest number of watermelons that the porter is willing to move along that trail. The porter may make multiple trips, but is not allowed to move more than this many melons overall. For example, the porter can move a total of 2 but not 3 watermelons from **B** to **C**.

(a) How should you instruct the porter to move the watermelons in order to maximise the number of them that reach the Canteen?

(b) Explain convincingly why it is not possible for any additional watermelons to make it to the canteen.