CSC 3130: Formal Languages and Automata Theory — Homework 4
The Chinese University of Hong Kong, Fall 2008 — due Friday 31 October

Each of the problems is worth 10 points. Please write your solutions clearly and concisely. If you do not explain your answer you will be given no credit. You are encouraged to collaborate on the homework, but you *must* write your own solutions and list your collaborators on your solution sheet. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please turn in the solutions by 11.59pm on Friday 31 October. The homework should be dropped off in the box labeled CSC 3130 on the 9th floor of SHB. Late homeworks will not be accepted.

In the context-free grammar descriptions below, the start variable is always the one on the left-hand side of the first production.

Problem 1

Draw pushdown automata for the following languages. Briefly describe how your automaton works.

(a) $L_1 = \{x\#y : x^R \text{ is a substring of } y,\ x, y \in \{a, b\}\}$, $\Sigma = \{a, b, \#\}$.

(b) $L_2 = \{a^i b^{2i+1} : i \geq 0\}$,

(c) $L_3 = \{a^m b^n c^p d^q : m + n = p + q\}$, $\Sigma = \{a, b, c, d\}$.

(d) $L_4 = \{a^i b^j : i > j\}$, $\Sigma = \{a, b\}$.

(e) $L_5 = \{x : x \text{ has the same number of } a\text{s and } b\text{s or same number of } b\text{s and } c\text{s.}\}$, $\Sigma = \{a, b, c\}$.

Problem 2

For each of the following languages, prove that it is not regular by using the pumping lemma for context-free languages. The alphabet is $\Sigma = \{0, 1\}$.

(a) $L_1 = \{www : w \in \{0, 1\}^n\}$.

(b) $L_2 = \{010^2 10^3 1 \cdots 0^n 1 : n > 0\} = \{01, 01001, 010010001, \dots\}$.

(c) $L_3 = \{0^n 1^{n^2} : n > 0\}$.

Problem 3

Consider the following context-free grammar G:

$$S \rightarrow \mathrm{a}S\mathrm{a} \mid \mathrm{b}.$$

(a) Write all items in this grammar.

(b) Construct an ϵ-NFA for the valid item updates.

(c) Convert the ϵ-NFA from part (b) to an NFA (without ϵ-transitions).

(d) Convert the NFA from part (c) to a DFA. (Your DFA should have 5 states, if you omit the "die" state.) Which of the states are shift states and which are reduce states?

(e) Using the DFA in part (d), show an execution of the LR(0) parsing algorithm on the input aabaa. Show the state of the stack, input, and DFA throughout the execution.

Problem 4

Context-free grammars are sometimes used to model natural languages. In this problem you will model a fragment of the English language using context-free grammars. Consider the following English sentences:

```
The girl is pretty.
The girl that the boy likes is pretty.
The girl that the boy that the clerk pushed likes is pretty.
The girl that the boy that the clerk that the girl knows pushed likes is pretty.
```

This is a special type of sentence built from a subject (`The girl`), a relative pronoun (`that`) followed by another sentence, a verb (`is`) and an adjective (`pretty`).

(a) Give a context-free grammar G that models this special type of sentence. Your terminals should be words or sequences of words like `pretty` or `the girl`.

(b) Is the language of G regular? If so, write a regular expression for it. If not, prove using the pumping lemma for regular languages.

(c) Can you give an example of a sentence that is in G but does not make sense in common English?