CSC 3130: Formal Languages and Automata Theory
The Chinese University of Hong Kong, Fall 2008

Homework 1
due Thursday 18 September

Each of the problems is worth 10 points. Please write your solutions clearly and concisely. If you do not explain your answer you will be given no credit. You are encouraged to collaborate on the homework, but you *must* write your own solutions and list your collaborators on your solution sheet. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please turn in the solutions by 11.59pm on Thursday 18 September. The homework should be dropped off in the box labeled CSC 3130 on the 9th floor of SHB. Late homeworks will not be accepted.

In the DFA and NFA transition tables below, $\rightarrow$ indicates the start state and $*$ indicates the accepting states.

Problem 1

Give a DFA for the following languages, specified by a transition diagram. For each one of them, give a short and clear description of how the machine works. Assume the alphabet is $\Sigma = \{0, 1, 2\}$:

(a) $L_1 = \{w \mid w \text{ is any string over } \Sigma \text{ that contains at least one '0'.}\}$

(b) $L_2 = \{w \mid w \text{ contains even number of 0s and an odd number of 1s.}\}$

(c) $L_3 = \{w = 0u12v \mid u, v \text{ are any strings over } \Sigma.\}$

Problem 2

This problem concerns the NFA given by the following transition table:

	0	1
$\rightarrow q_0$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\varnothing$	$\{q_1, q_2\}$
$*q_2$	$\varnothing$	$\varnothing$

Convert this NFA to a DFA using the method described in class. Specify the DFA by its transition diagram.

Problem 3

Consider the following languages over $\Sigma = \{0, 1\}$.

- L_1 is the language described by $(0 + 1)^*$.
- L_2 is the language of all strings that do not contain the pattern 00.
- L_3 is the language of the DFA given by the following transition table:

	0	1
$\rightarrow *q_0$	q_1	q_0
$*q_1$	q_2	q_1
$*q_2$	q_3	q_2
q_3	q_3	q_3

- L_4 is the language described by $\varepsilon + 0 + 1 + (\varepsilon + 0 + 1)(\varepsilon + 0 + 1)^*(\varepsilon + 0 + 1)$.
- L_5 is the language of all strings that have at most two 0s.
- L_6 is the language of the NFA given by the following transition table:

	0	1
$\rightarrow *q_0$	$\{q_0, q_1\}$	$\{q_0\}$
$*q_1$	$\{q_2\}$	$\varnothing$
$*q_2$	$\varnothing$	$\varnothing$

- L_7 is the language described by $1^*(011^*)^* + 1^*(011^*)^*0$.

Which of these languages are the same and which are different? To show two languages are the same give a short proof, and to show two languages are different give a string that is in one but not by the other. (You must provide an explanation to get credit.)

Problem 4

This problem concerns languages over the alphabet $\Sigma = \{1\}$. For any two integers $q, r \geq 0$, define the language $L_{r,q}$ over Σ as

$$L_{r,q} = \{1^{mq+r} : m \geq 0\} = \{1^q, 1^{q+r}, 1^{2q+r}, \dots\}.$$

For instance, $L_{1,3} = \{1, 1111, 1111111, \dots\}$.

(a) Show that for every q and r, the language $L_{r,q}$ is regular.

(b) Show that if L is a regular language over Σ, then L can be written as

$$L = L_{r_1,q} \cup L_{r_2,q} \cup \cdots \cup L_{r_k,q} \cup L_0,$$

where $0 \leq r_1 < \cdots < r_k$ are integers and L_0 is a finite set.