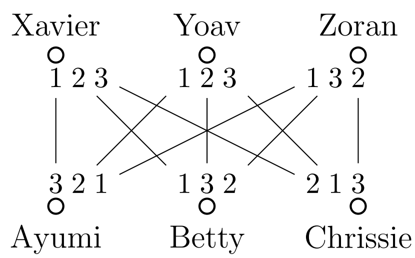
1. Write the proposition "There is a cycle of length 3" using logical connectives and quantifiers. Use symbols x, y, z for vertices and $E(x, y)$ for "$\{x, y\}$ is an edge."

2. Show that for all positive integers m and n, if $\gcd(m^2, n^2)$ is odd then m and n are not both even.

3. Use induction to show that for every $n \geq 1$, the $(n+1) \times n$ grid can be tiled using *two* sets of the following tiles: $1 \times 1, 1 \times 2, \ldots, 1 \times n$.

4. You start with the numbers $1, 2, 3, 4$. At each step, you take any three numbers a, b, c and replace them with $(a+b)/2, (b+c)/2, (c+a)/2$. The fourth one stays the same. Can you ever get $2, 3, 4, 5$?

5. Give a stable matching for the following preferences. Show that there is no rogue couple among the six man-woman pairs that are not matched to one another.



6. You are given a graph with 9 men and 9 women as vertices and all possible 81 man-woman pairs as edges. Let Ξ be any matching in this graph. Remove the edges in Ξ (but not the vertices.) Show that the remaining graph has a perfect matching.