Each question is worth 10 points. Please explain your solution clearly and concisely.

1. Write the proposition "There is at most one ball in every urn" using logical connectives and quantifiers. Use the symbols b_1, b_2 for balls, u_1, u_2 for urns and $IN(b, u)$ for "ball b is in urn u".

2. Show that for every two integers m and n, $(m+n)^3$ is even if and only if $m^3 + n^3$ is even.

3. A *cut-edge* in a connected graph is an edge e such that if e was removed, the graph would no longer be connected. Show that any connected graph in which all vertices have even degree does not have a cut-edge.

4. The graph G_1 consists of a single vertex. For $n \geq 1$, the graph G_{n+1} consists of two disjoint copies of G_n and a matching between the vertices of the two copies. How many edges does G_n have?

5. Let $f(n) = 1 + 1/3 + 1/5 + \cdots + 1/(2n-1)$. Show that f is $\Theta(\log n)$.

6. You drop 30 balls into 7 urns. Some of the balls are red and some are blue. Show that at least three balls of the same colour land in the same urn.

7. You are dealt 5 random cards from a 52-card deck. What is the probability that the largest face value is a 9? (The face values from smallest to largest are 2 3 4 5 6 7 8 9 10 J Q K A.)

8. You have overhang blocks 10, 11, up to n units long, one of each kind. They are stacked over the table from smallest to largest so that their left edges align. (See diagram for $n = 13$). Show that the configuration is not stable when n is sufficiently large.