1. Students from two Universities participated the same contest. To estimate their performance, 80 students from University A are sampled and their average score is 84; 100 students from University B are sampled, and their average score is 80. Assume the scores are independent normal random variables.

   (a) If the actual standard deviations are $\sigma_A = 12.4$ and $\sigma_B = 11.2$, what is the p-value for the hypothesis "University A students outperform University B students?"

   (b) If the adjusted sample deviations are $S_A = 12.4$ and $S_B = 11.2$, while the actual standard deviations are identical but unknown, what is the p-value for the hypothesis in (a)?

2. The dataset AUTO83B.DAT[1] contains 480 ceramic strength measurements for two batches of material. Assume the data consists of independent normal samples.

Batch	Samples	Mean	SD
1	240	688.9987	65.54909
2	240	611.1559	61.85425

   (a) What is the p-value for the hypothesis that the batches have different mean strengths? Assume the actual standard deviations are the same.

   (b) What is the p-value for the hypothesis that the batches have different standard deviations?

3. A cookie manufacturer wants to test if replacing milk chocolate with dark chocolate in their product will lower the calorie count. To do so it creates sixteen cookie batches, tests the samples, and obtains the following numbers:

with dark chocolate	113	120	138	120	100	118	138	123
with milk chocolate	138	116	125	136	110	132	130	110

   (a) If the batches were produced by eight (independent) cooks, each of which made one with dark and one with milk chocolate, which test would be appropriate to use, and what is the p-value?

   (b) If one cook produced all the dark chocolate batches and another one produced all the milk chocolate ones, which test would you use? How does the p-value change?

4. You are given two samples X_1, X_2 of a Uniform$(0, \theta_1)$ random variable and two samples Y_1, Y_2 of a Uniform$(0, \theta_2)$ random variable, all independent. You want to test the hypothesis $\theta_1 < \theta_2$ against the null hypothesis $\theta_1 = \theta_2$. Consider the test T that outputs + if $\min\{Y_1, Y_2\} > \max\{X_1, X_2\}$ and − if not.

   (a) What is the false positive error of T?

   (b) What is the false negative error of T assuming $\theta_2 \geq 2\theta_1$?

[1] https://www.itl.nist.gov/div898/handbook/eda/section3/eda3531.htm