

Each question is worth 10 points. Please explain your solution clearly and concisely.

1. Is the following deduction rule valid?

$$\frac{P \rightarrow Q \quad P \rightarrow R}{Q \text{ OR } R}$$

2. Let x be a real number. Show that if $x^2 + 5x < 0$ then $x < 0$.
3. Show that for every integer $n \geq 1$, $1 + 1/4 + 1/9 + \dots + 1/n^2 \leq 2 - 1/n$.
4. Show that for all integers x, y that are not both zero, $2x^3 + y^3 \neq 0$.
5. Do there exist integers s and t such that $11s + 9t = 1$? (Provide a proof.)
6. Alice stands one step West from Bob. In every round both of them move North, South, East, or West, at the same time, one step at a time. Can they ever meet? (Provide a proof.)