Each question is worth 10 points. Please explain your solution clearly and concisely.

1. Write the proposition "Every pair of strangers has a common friend" using connectives and quantifiers. Use $F(x, y)$ for "x is friends with y." (Two people are strangers if they are not friends.)

2. Let a and b be real numbers. Show that if a is rational and ab is irrational, then b is irrational.

3. Show that for every $\ell \geq 3$, a cycle of length ℓ has a perfect matching if and only if ℓ is even.

4. On input n, d the Extended Euclid's Algorithm outputs integers s, t such that $s \cdot n + t \cdot d = \gcd(n, d)$. Assume that $\gcd(n, d) = 1$. Show that $\gcd(s, t) = 1$.

5. The multiplicative inverse of 3 modulo 23 is 8. The multiplicative inverse of 12 modulo 23 is 2. What is the multiplicative inverse of 13 modulo 23? Explain your reasoning.

6. Show that for every $n \geq 2$, a 6 by n grid can be tiled using 2 by 1 L-shaped tiles.