1. Are the propositions "Every two people have a common friend" and "Every person has at least two friends" logically equivalent? Justify your answer.

2. Show that for every real number x, at least one of the numbers x, $x + \sqrt{2}$ is irrational.

3. Alice has an infinite supply of \$4 stamps and exactly three \$7 stamps. Can she obtain all integer postage amounts of \$18 and above? Justify your answer.

4. Show that for every integer $n \geq 1$, $1 + 1/4 + 1/9 + \cdots + 1/n^2 \leq 2 - 1/n$.

5. There are 7 girls and 15 boys in a group. Show that some girl is friends with at least five boys or some boy is friends with at most one girl (or both).

6. n white pegs and n black pegs are arranged in a line. In each step you are allowed to move any peg past *two* consecutive pegs of the opposite colour, left or right. Initially all white pegs are to the left of the black ones. Show that the colours can be reversed if and only if n is even.