Let P be a random variable that is equally likely to take the values $1/3$ and $2/3$. Let X be the number of heads when a coin with probability of heads P is tossed 10 times. Find $\mathrm{Var}[\mathrm{E}[X|P]]$.

Solution: The variance of P is $\frac{1}{2} \cdot (\frac{1}{3} - \frac{1}{2})^2 + \frac{1}{2} \cdot (\frac{2}{3} - \frac{1}{2})^2 = \frac{1}{36}$, so

$$\mathrm{Var}[\mathrm{E}[X|P]] = \mathrm{Var}[10P] = 100 \cdot \mathrm{Var}[P] = \frac{100}{36} = 2\tfrac{7}{9}.$$