

Judy's traffic time to school is between 15 and 20 minutes if the day is sunny, and between 20 and 25 minutes if the day is rainy, with all times being equally likely in each case. Assume that a day is sunny with probability  $2/3$  and rainy with probability  $1/3$ . What is the PDF of the traffic time, viewed as a random variable  $X$ ?

**Solution:** Let  $X$  be Judy's traffic time and  $Y$  be an indicator for rain (0 when it is sunny and 1 when it rains). The conditional PDF  $f_{X|Y}(x|0)$  is Uniform(15, 20) so it has value  $1/5$  when  $x$  is between 15 and 20 and zero otherwise. Similarly  $f_{X|Y}(x|1)$  has value  $1/5$  when  $x$  is between 20 and 25 and zero if not. By the total probability theorem,

$$f_X(x) = \frac{2}{3} \cdot f_{X|Y}(x|0) + \frac{1}{3} \cdot f_{X|Y}(x|1)$$

and so

$$f_X(x) = \begin{cases} \frac{2}{15}, & \text{if } 15 \leq x < 20, \\ \frac{1}{15}, & \text{if } 20 \leq x < 25, \\ 0, & \text{otherwise.} \end{cases}$$