

You flip two fair coins. If they both come out heads you stop. If not, you try again until they do. Let F be the total number of coin flips you performed. For example if the outcome is THHTHH then $F = 6$. What is the PMF (probability mass function) of F ?

Solution: F can never be odd as you always perform an even number of flips. To perform a total of $2k$ flips, the first $k - 1$ rounds must have all resulted in failure and the last one in success. The probability of each round succeeding is $1/4$ and the successes are independent of one another, so

$$P(F = 2k) = (3/4)^{k-1}(1/4),$$

where k ranges over $0, 1, 2, \dots$, or if you prefer

$$P(F = f) = (3/4)^{f/2-1}(1/4),$$

where f ranges over the nonnegative even numbers.