There are two coins. Coin A is fair: It has heads on one side and tails on the other. Coin B has heads on both sides. You choose a coin at random, flip it twice, and observe two heads. What is the probability that you chose coin A?

**Solution:** Let $A$ be the event I chose the first coin and $H$ be the event of two consecutive heads. Bayes' rule says that

$$\mathrm{P}(A|H) = \frac{\mathrm{P}(H|A)\,\mathrm{P}(A)}{\mathrm{P}(H|A)\,\mathrm{P}(A) + \mathrm{P}(H|A^c)\,\mathrm{P}(A^c)} = \frac{\mathrm{P}(H|A)}{\mathrm{P}(H|A) + \mathrm{P}(H|A^c)}$$

because $A$ and $A^c$ have (unconditional) probablity $1/2$. Since $\mathrm{P}(H|A) = 1/4$ and $\mathrm{P}(H|A^c) = 1$, we get that $\mathrm{P}(A|H) = 1/5$.