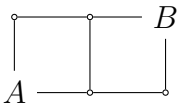
ENGG 2430A / ESTR 2002: Probability and Statistics for Engineers
The Chinese University of Hong Kong, Spring 2019 Practice Midterm 2

Each question is worth ten points. To receive full credit for your answer, you must clearly describe the sample space, the event of interest, and explain your calculations.

1. 3 red balls and 3 blue balls are randomly arranged on a line. Let X be the position of the first blue ball. (E.g. for the arrangement `RBRBBR`, $X = 2$.) Find the probability mass function of X.

2. Computers A and B are linked through seven cables as in the picture. Each cable fails with probability 10% independently of the others. What is the probability there is a connection between A and B?



3. Toss a coin 4 times. Let X, Y and Z be the number of heads among the first two, middle two, and last two tosses, respectively. Are X and Z independent given that $Y \neq 1$? Justify carefully.

4. The average lifetime of a lightbulb is 10 months. You install 10 lightbulbs today. What is the probability that at least one of them fails within a month? Assume their failures are independent.

5. Eight people's hats are mixed up and randomly redistributed. What is the expected number of pairs that exchanged hats (Alice got Bob's and Bob got Alice's)?