Each question is worth ten points. To receive full credit for your answer, you must clearly describe the sample space, the event of interest, and explain your calculations.

1. A coin is tossed 4 times. What is the probability there are at least 3 consecutive heads?

2. When a coin comes out heads, you win \$1. When it comes out tails, you lose \$2. You toss the coin twice. Find the probability mass function and the expected value of your profit.

3. Half the students know the answer to a true-false question. The other half guesses at random. I ask a random student and his answer is correct. What is the probability he knows the answer?

4. An unknown number of independent trials is performed, each of which succeeds with the same probability. You can only observe the number of successful trials. After many runs of this experiment you conclude that the expected number of successful trials is 6, and the variance of this number is 2. How many trials are performed?

5. 15 runners divided into 3 teams are to participate in a race. If a runner wins, everyone on his team gets a \$1 prize. Your objective is to minimize the expected amount of prize money given away. How do you divide the runners into teams? Assume each runner is equally likely to win.