Practice questions

1. A point is chosen uniformly at random inside a triangle with base 1 and height 1. Let X be the distance from the point to the base of the triangle. Find the CDF and the PDF of X. (*Textbook problem 3.2.5*)

2. The arrival times of the 193 ENGG 2430A / ESTR 2004 to class are normal random variables with a mean value of 9.25am and a standard deviation of 5 minutes.

   (a) What is the expected number of students that have arrived by 9.30am?

   (b) Assuming students' arrivals are independent, what is the probability that everyone has made it by 9.45am?

3. Three points are dropped at random on the perimeter of a circle with 1 unit circumference.

   (a) What is the probability that they all fall within 1/4 of a unit of one another?

   (b) What is the probability that every pair of them is at least 1/4 of a unit apart? (**Hint:** Fix one of the three points.)

4. A coin has probability P of being heads, where P itself is a Uniform$(0, 1)$ random variable. The coin is flipped twice. Given that it comes out heads both times, what is the (posterior) PDF of P? What is its expected value?

5. Here is a way to solve Buffon's needle problem without calculus. Recall that an ℓ inch needle is dropped at random onto a lined sheet, where the lines are one inch apart.

   (a) Let A be the number of lines that the needle hits. Let B be the number of times that a polygon of perimeter ℓ hits a line. Show that $\mathrm{E}[A] = \mathrm{E}[B]$. (**Hint:** Use linearity of expectation.)

   (b) Assume that $\ell < \pi$. Calculate the expected number of times that a circle of perimeter ℓ hits a line.

   (c) Assume that $\ell < 1$. Use part (a) and (b) to derive a formula for the probability that the needle hits a line. (**Hint:** The number of hits is a Bernoulli random variable.)