Practice questions

1. You toss a coin 100 times. Which of the following random variables are independent?
   (a) The number of consecutive heads `HH` and the number of consecutive tails `TT`.
   (b) The number of consecutive heads in the first 50 tosses and the number of consecutive tails in the last 50 tosses.
   (c) The random variables in part (b), conditioned on having exactly 50 heads in the 100 coin tosses.

2. A fair coin is tossed 100 times. What is the expected number of times T that three consecutive heads occur? For example, if the outcome is `HHHHTHHH` then $T = 3$.

3. In 2017 there were 0.848 men for every woman in Hong Kong. Men and women had life expectancies of 81.7 years and 87.7 years, respectively. What was the life expectancy of a random person?

4. Consider $2m$ persons forming m couples who live together at a given time. Suppose that at some later time, the probability of each person being alive is p, independent of other persons. At that later time, let A be the number of persons that are alive and let S be the number of couples in which both partners are alive. Find $E[S \mid A]$. *(Textbook problem 2.32)*

5. Charlie is conducting telephone surveys as a part time job at CCPOS of CUHK. He needs 2 more surveys before going home. However, on randomly dialed calls, only 15% of receivers would complete the survey. Let X be the number of dials Charlie needs to make before going home. Find the expected value and variance of X.

Additional ESTR 2002 questions

6. Find the values of the following two-player games.
   (a) The game whose payoffs (the amount that Alice gives to Bob) are given in this table:

Alice \ Bob	b_1	b_2	b_3
a_1	3	-1	2
a_2	-1	2	-2

   (b) Alice's action is a subset A of $\{1, \ldots, n\}$. Bob's action is also a subset B of $\{1, \ldots, n\}$. The payoff is $(-\lambda)^{|A \ominus B|}$, where $0 \leq \lambda \leq 1$ is some constant and $A \ominus B = (A \cup B) \setminus (A \cap B)$ is the *symmetric set difference* of A and B.

7. Alice has two fair coins. Instead of heads and tails, coin A has side values 1 and -1. Coin B has values 2 and -2. Alice asks Bob to toss a coin 100 times. She says "heads" if the sum of all the observed values is positive, "tails" if it is negative, and declares failure if it is exactly zero. For each toss, Bob can decide which coin to toss based on the values observed so far.
   (a) Conditioned on no failure occurring, what is the probability that Alice says "heads"?
   (b) What if, instead of tossing the coin 100 times, Alice stops as soon as the sum of the observed side values reaches either 100 or -100?