1. In how many ways can we roll 4 dice so that

   (a) The face values of the dice are all different?

   Solution: The first die has 6 possible outcomes. For each of them, there are 5 possibilities for the second die that are different from the first, 4 possibilities for the third die different from the first two, and 3 possibilities for the last die different from the first 3. The total number of possibilities is therefore $6 \cdot 5 \cdot 4 \cdot 3 = 360$.

   (b) The face values of the dice are increasing (e.g., 2356 but not 3516, 1224)?

   Solution: We are choosing 4 distinct face values out of the set $\{1, 2, 3, 4, 5, 6\}$ and then writing them down from smallest to largest. This can be done in $\binom{6}{4} = 15$ possible ways.

2. A bin contains 10 black balls and 10 white balls. You draw three balls without replacement. What is the probability that all three are black?

   Solution: The sample space Ω consists of all $\binom{20}{10}$ arrangements of the balls. We assume equally likely outcomes. The event A consists of those arrangements in which three black balls appear in the first three positions. As the other 7 black balls can appear anywhere in the other 17 positions, A has size $\binom{17}{7}$. By the equally likely outcomes formula,

$$\mathrm{P}(A) = \frac{\binom{17}{7}}{\binom{20}{10}} = \frac{10 \cdot 9 \cdot 8}{20 \cdot 19 \cdot 18} \approx 0.1053.$$

3. ENGG 2760A has 100 students this year, including Alice and Bob. The students are randomly divided into three tutorials with 30, 35, and 35 students, respectively.

   (a) What is the probability that Alice and Bob are both assigned to the 30-student tutorial?

   Solution: The sample space Ω consists of all possible *partitions* of the 100 students into three tutorials (subsets) of size 30, 35, and 35. There are $\binom{100}{30,35,35}$ such partitions. The event A_1 of interest consists of those partitions in which both Alice and Bob land in the 40-student tutorial. The size of A_1 is the number of ways to partition the rest of the students into the remaining tutorial slots, which is $\binom{98}{28,35,35}$. Therefore

$$\mathrm{P}(A_1) = \frac{\binom{98}{28,35,35}}{\binom{100}{30,35,35}} = \frac{30 \cdot 29}{100 \cdot 99} \approx 0.0879.$$

   (b) What is the probability that Alice and Bob are assigned to the same tutorial?

   Solution: Now the event A of interest is a union of three disjoint events A_1, A_2, and A_3 consisting of those outcomes in which Alice and Bob are assigned together into the first, second, and third tutorial, respectively. By a similar calculation as in part (a), $|A_2| = |A_3| = \binom{98}{30,33,35}$. As $|A| = |A_1| + |A_2| + |A_3|$ we get that

$$\mathrm{P}(A) = \frac{|A|}{|\Omega|} = \frac{\binom{98}{28,35,35} + 2 \cdot \binom{98}{30,33,35}}{\binom{100}{30,35,35}} = \frac{30 \cdot 29 + 2 \cdot 35 \cdot 34}{100 \cdot 99} \approx 0.3283.$$

4. A six-sided die is rolled three times. Which is more likely: A sum of 11 or a sum of 12? *(Textbook problem 1.49)*

   Solution: Let A and B be the events of a sum of 11 and a sum of 12, respectively. As the outcomes are equally likely, the probabilities of the two sums are $|A|/6^3$ and $|B|/6^3$ so we need to determine which of the sets A and B is bigger. The set A can be partitioned into A_1 up to A_6 depending on the first die roll. Similarly, B can be partitioned into B_1 up to B_6. Now A_1 has the same size as B_2 as they both share the same pairs of values for the second and third dice. By the same argument, $|A_2| = |B_3|$, $|A_3| = |B_4|$, $|A_4| = |B_5|$, and $|A_5| = |B_6|$. Comparing $|A|$ and $|B|$ therefore amounts to comparing $|A_6|$ and $|B_1|$. For an outcome to be in A_6 the remaining two rolls must add up to 5, while for B_1 they must add up to 11. Therefore $|A_6| = 4$ and $|B_1| = 2$, so A is the larger set and a sum of 11 is more likely.