ENGG 2760A / ESTR 2018: Probability for Engineers

Tutorial 7

1. Quiz 5
2. Quiz 6
3. Review

Quiz 5:

A five person committee is chosen at random from 5 girls and 15 boys. Conditioned on there being at least one girl on the committee, what is the expected number of boys?

A

$|A| = \binom{5}{1}\binom{19}{4} > |\Omega| = \binom{20}{5} = \frac{20 \times 19 \times 18 \times 17 \times 16}{5 \times 4 \times 3 \times 2 \times 1}$

$= 4\binom{19}{4}$

Alice, Amy

Alice Amy. XXX

Amy Alice, XXX

repeated calculation

A: At least 1 girl

$P(A) = 1 - P(A^c)$

$P(X=0)$

no girl on the committee

Quiz 6:

A train's arrival time at Kowloon station is T minutes past noon, where the PDF of T is

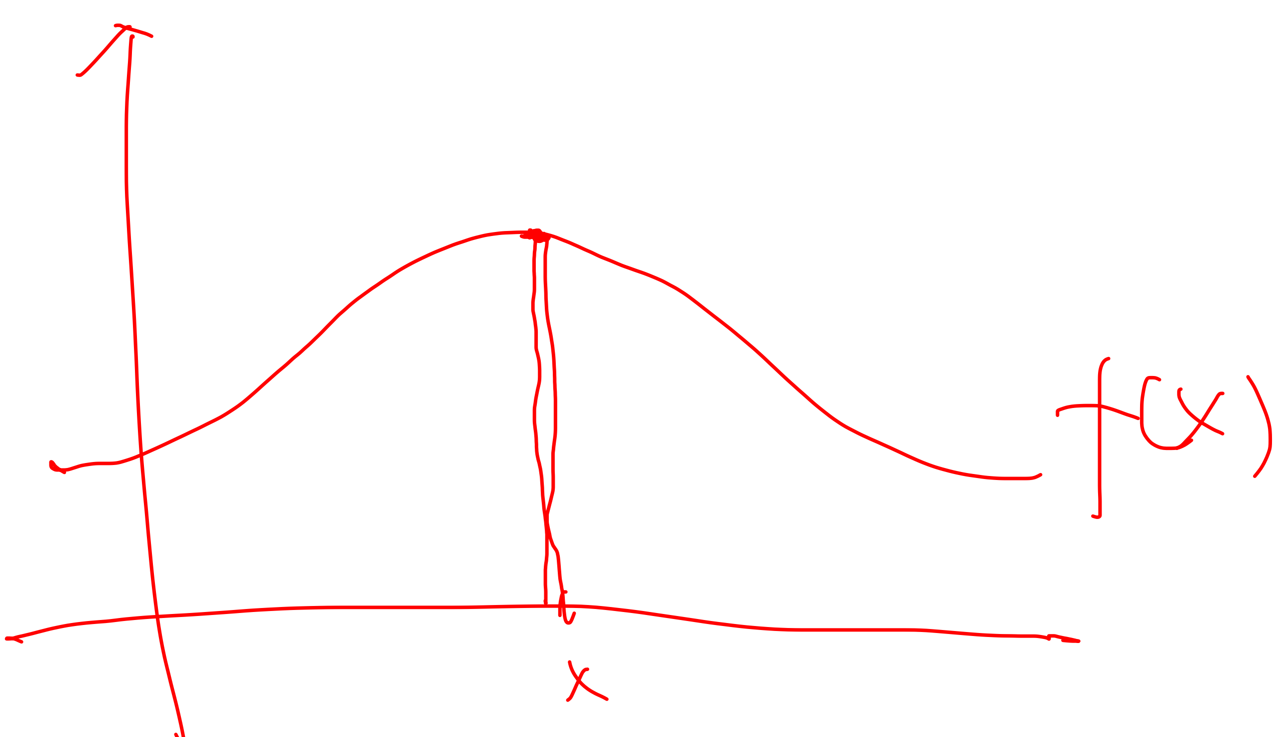
$$f_T(t) = \begin{cases} 1/(t+1)^2, & \text{if } t \ge 0 \\ 0, & \text{otherwise.} \end{cases}$$

5/11

Given that the train hasn't arrived by 12:05, what is the probability that it arrives by 12:10?

$$P(T\le 10 \mid T>5) = \frac{P(10\ge T>5)}{P(T>5)} = \frac{F_T(10)-F_T(5)}{1-F_T(5)} = \frac{5}{11}$$

$$F_T(t)=\int_{-\infty}^{t} f_T(t)\,dt$$



$$P(X=x)=\lim_{\alpha\to 0} P(X\le x)-P(X\le x-\alpha)=0$$

Review

1. Probability & Counting

1) Ω 2) A 3) $P(A) = \frac{|A|}{|\Omega|}$ Equally likely outcomes

prevent the repeated calculation

Check!!!

2. Axioms of Probability

1) $P(A) \in [0,1]$

2) $P(\Omega) = 1$

3) if $E_1, E_2, \cdots$ disjoint, $P(E_1 \cup E_2 \cup \cdots) = P(E_1) + P(E_2) + \cdots$

3. Conditional Probability

$$\Rightarrow P(A|F) = \frac{P(A \cap F)}{P(F)}$$ ☆☆☆

$$\Rightarrow P(A_1 \cap A_2 \cap A_3 \cap \cdots) = P(A_1) P(A_1|A_2) P(A_3|A_1 \cap A_2) \cdots$$

multiplication

4. Independent

If A.B are independent $\Longleftrightarrow$ $P(A \cap B) = P(A)P(B)$

$\Downarrow$

$$P(A \cap B|F) = P(A|F)\, P(B|F)$$

5. Random Variable

1) Discrete~ vs Continous~

	Discrete~	Continous~
	PMF $f(x) = P(X=x)$	PDF $f(x) = \frac{dF}{dx}$
$P(X \le a)$	$\sum_{x \le a} f(x)$	$\int_{-\infty}^{a} f(x)\,dx = CDF = F(a)$
$E[X]$	$\sum_x x f(x)$	$\int_{-\infty}^{\infty} x f(x)\,dx$
$E[X^2]$	$\sum_x x^2 f(x)$	$\int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$Var[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

2) Joint PMF & Marginal PMF

Joint PMF

Y \ X	x_1	x_2	...
y_1			
y_2			
$\vdots$			

Marginal PMF

X	x_1	x_2	...
$P(X=x)$			

Y	y_1	y_2	...
$P(Y=y)$			

3) Conditional PMF

$$P(X=x \mid Y=y) = \frac{P(X=x \text{ and } Y=y)}{P(Y=y)}$$

6. Expectation

Test: For every two random variables,

(a) $E[X+Y] = E[X] + E[Y]$ T/F $\Rightarrow$ Linearity of expectation

(b) $E[XY] = E[X]E[Y]$ T/F

if X, Y are independent

1) Linearity of expectation + Indicator Variable

$\Rightarrow \quad X = X_1 + \cdots + X_n$

indicator varible $X_i = \begin{cases} 1 & \text{if happened} \\ 0 & \text{o/w} \end{cases}$ $E[X_i] = p(X_i)$

$\Rightarrow E[X] = E[X_1] + \cdots + E[X_n]$

Linearity of expectation $= p(X_1) + \cdots + p(X_n)$

2) Conditional Expectation

$\Rightarrow E[X|A] = \sum x\, p(x|A)$

$E[X] = E[X|A]\,p(A) + E[X|A^c]\,p(A^c)$