ENGG 2760A / ESTR 2018: Probability for Engineers

Tutorial 4

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$P(T \cap M^c) \cancel{=} P(T) \times P(M^c)$ T. M dependent

$P(T) = P(TM) + P(TM^c)$

$P(AB) = P(A)P(B)$ iff A. B are independent

$P(MT) = 60\%$

$P(M)P(T) = 70\% \times 80\% = 56\%$ $\Leftrightarrow$ M&T are dependent.

$P(T \mid B^c)$ — they attend both

She wasn't in class on Monday

$B^c \neq M^c$

$MT^c, \underbrace{TM^c, M^cT^c}$

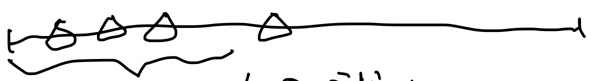
Quiz 3

At turn t, Alice has drawn the last Ace.

$\rightarrow \Omega : \{A_1, A_2, A_3, A_4 ;\ A_i \in (1,2,\cdots,52),\ A_i \neq A_j,\ i,j = 1 \cdots 4\}$

A_i : the position of i-th ace

$A_1\ A_2\ A_3 \quad A_4$



$t-1$ position $\quad$ t-position

$\boxed{52 \geq t \geq 4}$

$\rightarrow A : \{A_1, A_2, A_3, A_4 :\ A_i = t,\ A_j \in (1,2,\cdots,t-1)\}$

$A_i \neq A_j,\ i,j = 1,\cdots,4$

$|\Omega| = \binom{52}{4} \cdot 4!$

$|A| = \underbrace{\binom{4}{1}}_{\text{position } t} \underbrace{\binom{t-1}{3}}_{\text{other three ace}} \cdot 3!$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{\binom{4}{1}\binom{t-1}{3} 3!}{\binom{52}{4} \cdot 4!} = \frac{4(t-1)(t-2)(t-3)}{52 \times 51 \times 50 \times 49}$$

$t \in [4, 52]$

$t=5$

$t=4$

A A A A $\quad$ $p(A) = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49}$

1 2 3 4

$3!$ $\left\{ \begin{array}{l} \vdots \\ A_3 \quad A_2 \quad A_4 \\ A_2 \quad A_3 \quad A_4 \\ A_2 \quad A_4 \quad A_i \end{array} \right.$ $\quad A \rightarrow A_1, A_2, A_3, A_4$

1 2 3 4 5 $\leftarrow \binom{4}{1}$

A_2, A_3, A_4 (if 5-th position is A_1)

$\binom{4}{3}\ 3!$

sample space:

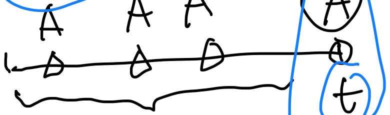
. 4 Ace

total position 52

$\binom{52}{4} \cdot 4!$

$t \geq x$

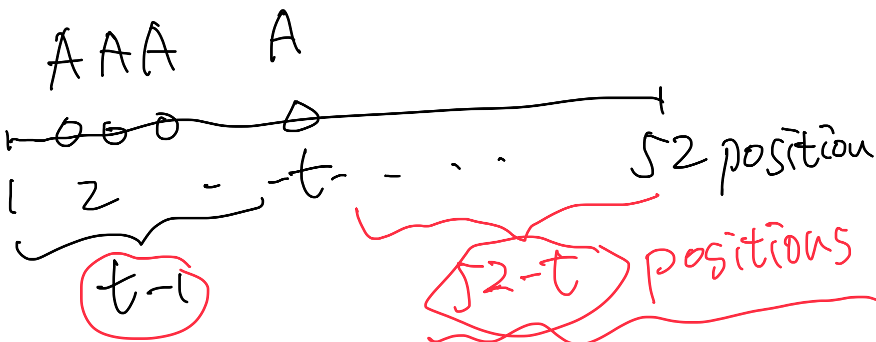
A A A $\quad$ A



$t-1 \quad t$

$\binom{t-1}{3} \cdot 3! \times \binom{4}{1}$

Second method



$\binom{48}{t-4}$ other cards. AAA in $1 \sim t-1$ position

$\binom{48}{t-4}\cdot(t-1)!$

$\binom{4}{1} \leftarrow A$ $\qquad$ t position

$(52-t)!$ $\qquad$ $t+1 \sim 52$ position

$$|A| = \binom{48}{t-4}(t-1)!\binom{4}{1}(52-t)!$$

$$|\Omega| = 52!$$

$$P(A) = \frac{\binom{48}{t-4}(t-1)!\binom{4}{1}(52-t)!}{52!} = \frac{48!\,(t-1)!\,4\,\cancel{(52-t)!}}{(t-4)!\,\cancel{(52-t)!}\,52!}$$

$$= \frac{4(t-1)(t-2)(t-3)}{52\times51\times50\times49} \qquad 4 \le t \le 52$$

HW5

Q2:

X \ Y	1	2	3	4
1	O	X	X	X
2	X	X	X	X
3	X	X	X	X
4	X	X	X	X

Z	1	2	3	4
$P_X(Z)$	O	X	X	X
$P_Y(Z)$	O	X	X	X

$E[X+Y]$
$= E[X] + E[Y] = 5$

Q3:

3 coins → p baised coin
3 coins → 2 fair coin

If p-baised coin is H (p), reports → 00 / 11, based on the result of fair coin

else T (1−p), reports → 00 / 01 / 10 / 11, base on two fair coins

X = first report

second report = Y

X \ Y	0	1
0	$p\cdot\frac{1}{2}+(1-p)\cdot\frac{1}{4}$	$(1-p)\cdot\frac{1}{4}$
1	$(1-p)\cdot\frac{1}{4}$	$p\cdot\frac{1}{2}+(1-p)\frac{1}{4}=\frac{1+p}{4}$

$P(X=0)=\frac{1+p}{4}+\frac{1-p}{4}=\frac{1}{2}$

$P(X=1)=P(Y=0)=P(Y=1)$
$=\frac{1}{2}$

Q4(a) 100 balls, 100 bins, balls are independent to land in any of bins

Let $X_i = \begin{cases} 1, & i\text{-th bin receives exactly one ball} \\ 0, & \text{others} \end{cases}$

$E[X_i] = 1 \times P(X_i=1) + 0 \times P(X_i \neq 1)$

$= P(X_i = 1)$

$X = X_1 + X_2 + \cdots + X_{100}$

$E[X] = E[X_1] + \cdots + E[X_{100}]$

$E[X_i] = P(X_i = 1)$ for i-th bin, we have 100 balls

$= \binom{100}{1} \cdot \frac{1}{100} \left(\frac{99}{100}\right)^{99}$ ← other 99 balls

← other 99 bins

Binomial(100, 1/100)

expected number of bins

$E[X] = 100 \times 100 \times \frac{1}{100} \times \left(\frac{99}{100}\right)^{99}$

$= \frac{99^{99}}{100^{98}} \simeq 36.973$

Q4(b)

Y: the number of balls are not alone in their bin.

Z: ——— of balls are alone ~.

$\boxed{X + Y = 100}$ → $E[X+Y] = E[X] + E[Y] = E[100] = 100$

One ball is alone in their bin ⟺ the bin just receives one ball

$E[X] = \frac{99^{99}}{100^{98}} \approx 36.973$ $E[Y] \approx 100 - 36.973 = 63.027$

HW 4 3 3-sided dice

Q1(c)

X	$3/3$	$4/3$	$5/3$	$6/3$	$7/3$	$8/3$	$9/3$
$P(avg=x)$	$1/3^3$	$3/3^3$	$6/3^3$		$6/3^3$		
	$\{1\,1\,1\}$	$1+1+2$	$1+1+3$ $1+2+2$ x_i		$3+3+1$ $3+2+2$ $4-x_i$		

$P(SUM) = P(12 - SUM)$

MAX $- 4 =$ MIN

Q4(a) HT, TH

HTH

THT

$T \geq 3$

The second flip at last position

The first flip have $t-2$ choices.

$\underbrace{H\,H\cdots HT\,T\cdots TH}_{t}$

$\times \quad t-2 \quad \times$

last position is H

$P(T=t) = P(T=t|H)P(H) + P(T=t|T)P(T)$

$= \dfrac{t-2}{2^{t-1}} \cdot \dfrac{1}{2} \times 2 = \dfrac{t-2}{2^{t-1}}$

Q4(b)

2 or 3 !

$\underbrace{}_{t-1}\ \underbrace{1}_{t}$

$T=t | L=1 : \{2,3\}^{t-1} - \{2\}^{t-1} - \{3\}^{t-1}$

$P(T=t|L=1) = \dfrac{2^{t-1}-2}{3^{t-1}}$

$(L=i)$

$P(T=t) = \sum^{3}_{i=1} P(T=t|L=i)P_{\wedge}$

$= 3 \times \frac{1}{3} \times \dfrac{2^{t-1}-2}{3^{t-1}} = \dfrac{2^{t-1}-2}{3^{t-1}}$