ENGG 2760A / ESTR 2018: Probability for Engineers

Tutorial 3

1. Quiz 1
2. Quiz 2
3. Homework 3

1. set vs the size of set

$P(A) = \frac{A}{\Omega}$ set X

$P(A) = \frac{|A|}{|\Omega|}$ ✓

$\underbrace{|A|}_{\text{the size of set A}} = \underbrace{\{(a, b. c)\}}_{\text{set}} = 1$

the size of set A

number ≥ 0

set

collection of elements

2. check carefully

Quiz 2:

T 80%

60%

$\Omega = \{ \underbrace{MT, MT^c}_{M}, M^cT, M^cT^c \}$

10% 20%

$P(M) = 70\%$

$= P(T) - P(MT)$

$$P(T|M^c) = \frac{P(TM^c)}{P(M^c)} = \frac{P(TM^c)}{1 - P(M)} = \frac{20\%}{1-70\%} = \frac{2}{3}$$

hl:

(a) $\Omega = \{5R, 5B, 5G\}$ $\quad A = \{\text{draw a red ball}\}$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{5}{5+5+5} = \frac{1}{3}$$

(b) same as (a)

(c) $\Omega = \{4R, 5B, 5G\}$ $\quad A = \{\text{draw a red ball}\}$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{4}{4+5+5} = \frac{2}{7}$$

not blue

(d) $\Omega = \{4R, 5B, 5G\}$ $\quad A = \{\text{draw a red or green ball}\}$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{9}{14}$$

(e) $\Omega = \{ (X_1, X_2) : X_1, X_2 \in \{5R, 5B, 5G\} \}$

$$P(2R \mid 1B^c) = \frac{P(2R \cap 1B^c)}{P(1B^c)}$$

$$P(1B^c) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(2R \cap 1B^c)$$

$$A = \{(R,R), (G,R)\}$$

$$P(A) = \frac{5}{15} \times \frac{4}{14} + \frac{5}{15} \times \frac{5}{14}$$

$$P(2R \mid 1B^c) = \frac{5\times4+5\times4}{15\times14} \Big/ \frac{2}{3} = \frac{9}{28}$$

2. A, B are independent $\iff P(A\cap B) = P(A)\cdot P(B)$

$$P(E_{12}) = P(E_{23}) = P(E_{13}) = \frac{6^2}{6^3} = \frac{1}{6}$$

(a) $P(\underbrace{\overset{1=2}{E_{12}} \cap \overset{1=3}{E_{13}}}_{1.2.3 \text{ all same}}) = \frac{6}{6^3} = \frac{1}{6^2} = P(E_{12})\cdot P(E_{13})$ True

(b) $P(E_{12}\cap E_{13}\cap E_{23}) = \frac{1}{6^2} \neq \underline{P(E_{12})P(E_{13})P(E_{23})} = \frac{1}{6^3}$ False,

(c) $\underbrace{P(E_{12} \cap E_{23} | E_{13})}$ $\overset{?}{=}$ $P(E_{12}|E_{13}) P(E_{23}|E_{13})$

$$P(E_{12} \cap E_{23} | E_{13}) = \frac{P(E_{12} \cap E_{23} \cap E_{13})}{P(E_{13})} = \frac{6^{-2}}{6^{-1}} = \frac{1}{6}$$

$$P(E_{12}|E_{13}) = \frac{P(E_{12} \cap E_{13})}{P(E_{13})} = \frac{6^{-2}}{6^{-1}} = \frac{1}{6}$$

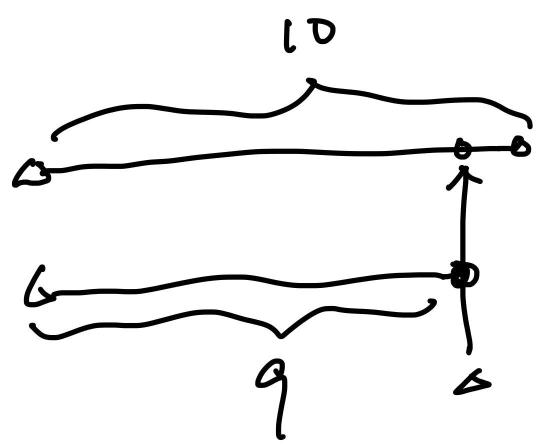
$$P(E_{23}|E_{13}) = \frac{1}{6}$$

False

$$P(E_{12}|E_{13}) P(E_{23}|E_{13}) = \frac{1}{6^2} \neq P(E_{12} \cap E_{23} | E_{13}) = \frac{1}{6}$$

3. Alice : 10 coin

Bob : 9 coin



After A & B have flipped first 9 coins

$\left\{ \begin{array}{l} A: \{\text{Alice more heads}\} \\ B: \{\text{Bob more heads}\} \\ S: \{\text{A \& B same number of heads}\} \end{array} \right.$

$\downarrow$

$P(A) + P(B) + P(S) = 1$

$P(A) = P(B)$

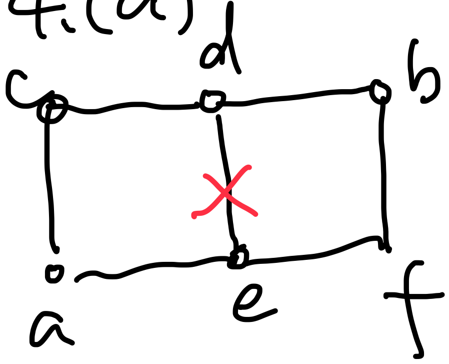
$$P(E) = P(E|A)P(A) + P(E|B)P(B) + P(E|S)P(S)$$

$E: \{\text{Alice gets more heads than Bob}\}$

$$\left. \begin{array}{l} P(E|A) = 1 \\ P(E|B) = 0 \\ P(E|S) = \frac{1}{2} \end{array} \right\} \begin{aligned} P(E) &= P(A) + \frac{1}{2} P(S) \\ &= P(A) + \frac{1}{2}(1 - P(A) - P(B)) \\ &= \frac{1}{2} + \frac{1}{2}\underbrace{(P(A) - P(B))}_{=0} = \frac{1}{2} \end{aligned}$$

$\hookrightarrow$ Alice flipped head in 10th coin

4.(a)



F: {de fails} C: {a connection a→b}

$$P(C|F)=P(T\cup B)=P(T)+P(B)-P(T\cap B)$$

$$C|F=\{\underbrace{a\to c\to d\to b}_{T},\ \underbrace{a\to e\to f\to b}_{B}\}$$

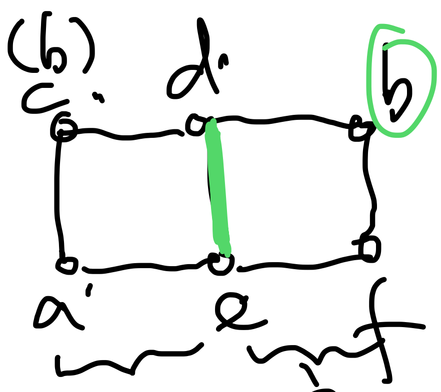
$P(T)=0.9^3$

$P(B)=0.9^3$

$\to P(C|F)=0.9^3+0.9^3-0.9^6=2\times0.9^3-0.9^6$

T & B are independent $P(T\cap B)=P(T)\cdot P(B)=0.9^6$

(b)



$P(C|F^c)$ ← condition F^c: {de doesn't fail}

L: {a→e, a→c→d} indep } independent

R: {d→b, e→f→b}

a→d L or a→e

R d→b or e→b

$$P(C|F^c)=P(L\cap R|F^c)=P(L|F^c)*P(R|F^c)$$

$$P(L|F^c)=P(a\to e)+P(a\to c\to d)-P(a\to e)P(a\to c\to d)$$

$$=0.9+0.9^2-0.9^3$$

$$P(R|F^c)=P(L|F^c)$$

$$\Rightarrow P(C|F^c)=(0.9+0.9^2-0.9^3)^2$$

(c) $P(C)=P(C|F)\,P(F)+P(C|F^c)\,P(F^c)$

$= \_\_\_\_ \times 0.9 + \_\_\_\_ \times 0.1$