Tutorial 2

Quiz 1:

$\Omega = \{1, 2, 3, \cdots, 6\}^3 \quad |\Omega| = 6^3$

$A = \{(a, b, c) : c > a \text{ and } c > b\}$

a, b, c : the face value of Alice, Bob, Charlie

$A = A_1 \cup A_2 \cup \cdots \cup A_6$

A_i : charlie's face value is $\underline{i}$

$\Rightarrow |A_i| = (i-1)^2 = \underbrace{(i-1)}_{\#a \text{ possible choice}} \times \underbrace{(i-1)}_{\#b \text{ possible choice}}$

$|A| = |A_1| + |A_2| + \cdots + |A_6|$

$= 0 + 1^2 + 2^2 + \cdots + 5^2 = 55$

Assume outcomes equally likely.

$P(A) = \frac{|A|}{|\Omega|} = \frac{55}{216} \simeq 0.255$

HW

Q1: $\Omega = \{AB, AC, BC\}$

A, B, C: Alice, Bob, Charlie get one of tickets

$P(\Omega) = 1 = P(AB) + P(AC) + P(BC)$ ①

$A = \{AB, AC\}$

$P(A) = P(AB) + P(AC) = 60\%$ ②

$B = \{AB, BC\}$

$P(B) = P(AB) + P(BC) = 70\%$ ③

$\Rightarrow$ ② + ③

$P(A) + P(B) = P(AB) + P(AC) + P(AB) + P(BC)$

$60\% + 70\% = P(AB) + P(\Omega)$

$\Rightarrow P(AB) = 60\% + 70\% - 1 = 30\%$

$1 = 100\%$

$$P(AB) = P(A)\cdot P(B|A) \neq P(A)\cdot P(B)$$

A, B are independent

Method 2:

$P(A^c)$: the probability of event BC

$= 1 - P(A) = 1 - 60\% = 40\%$

$P(B^c) = 1 - P(B) = 1 - 70\% = 30\%$

$\Omega: \{AB, AC, BC\} = \{C^c, B^c, A^c\}$

$P(\Omega) = P(C^c) + P(B^c) + P(A^c) = 1$

$\Rightarrow P(C^c) = 1 - P(A^c) - P(B^c) = 1 - 40\% - 30\%$

$P(AB) = 30\%$

Q2:

G: the quality of the parts is good

SD: ~ slightly defective } get slipped

OD: ~ obviously defective, discard

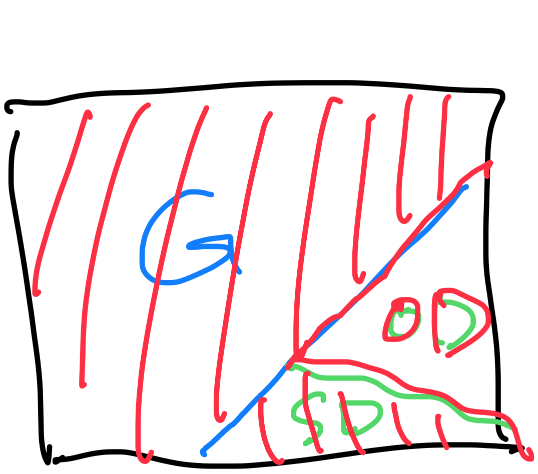
$P(G)=90\%$ $P(SD)=2\%$ $P(OD)=8\%$

$$\Rightarrow P(G|OD^c) = \frac{P(G \cap OD^c)}{P(OD^c)} = \frac{P(G)}{1-P(OD)}$$

$$= \frac{90\%}{1-8\%} \approx 0.978$$

$$\Rightarrow P(SD|OD^c) = \frac{\boxed{P(SD \cap OD^c)}}{P(OD^c)} = \frac{P(SD)}{1-P(OD)}$$

$$= \frac{2\%}{1-8\%} \approx 0.022$$



$\Omega = \{G, SD, OD\}$

$P(G) = 90\%$ $P(SD) = 2\%$

$P(OD) = 80\%$

$SD \cap OD^c$

$\|$

$SD \cap (G \cup SD) = SD$

Q3:

$\Omega = \{H, T\}^5 \quad |\Omega| = 2^5$

(a) M: a majority of the flips are heads

(at least 3 out of 5)

$M = H_3 \cup H_4 \cup H_5$

H_i: the number of heads is i

$|M| = |H_3| + |H_4| + |H_5|$

$= C_3^5) + C_4^5) + \binom{5}{5} = 16$

Assume outcomes equally likely.

$P(M) = \frac{|M|}{|\Omega|} = \frac{16}{2^5} = \frac{1}{2}$

(b) $P(M|H_1)$

$$= \frac{P(M \cap H_1)}{P(H_1)}$$

$P(H_1)$

H_1: first flip is a head $\quad P(H_1) = \frac{1}{2}$

H

$\Omega = \{H, T\}$

$P(M \cap H_1)$

$M \cap H_1$: a majority of the flips are heads and first is head

at least 2 out of 4

$$|M \cap H_1| = C^4_2 + \binom{4}{3} + \binom{4}{4} = 11$$

$$P(M \cap H_1) = \frac{11}{2^6}$$

$$P(M|H_1) = \frac{11}{16}$$

(c) $P(H_1 | M) = \dfrac{P(M \cap H_1)}{P(M)} = \dfrac{11/26}{1/2} = \dfrac{11}{16}$

Q4:

E_i: the first ball is drawn in the i-th turn.

E: Alice gets the white ball

$$E = E_1 \cup E_3 \cup E_5 \cancel{\cup E_7}$$

$$P(E_1) = \frac{3}{8}$$

$P(B_2|B_1)$

$$P(E_3) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \leftarrow P(W_3|B_1 \cap B_2)$$

3rd: W

1st: Black 2nd: B

$P(B_4|B_1B_2B_3)$

$$P(E_5) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{3}{4} \leftarrow P(W_5|B_1B_2B_3B_4)$$

$P(B_1)$ $P(B_2|B_1)$ $P(B_3|B_1B_2)$

$$P(E) = P(E_1) + P(E_3) + P(E_5) \approx 0.607$$

HW1:

Q2:

$B_1B_2B_3$: draw three balls are all black

$$P(B_1B_2B_3) = P(B_1)P(B_2|B_1)P(B_3|B_1B_2)$$
$$= \frac{10}{20} * \frac{9}{19} * \frac{8}{18} = \frac{10\times9\times8}{20\times19\times18}$$

Q3:

(a) A_1: Alice in the 30-student t~

B_1: Bob ~ 30-student ~

$$P(A_1 \cap B_1) = P(A_1)\cdot P(B_1|A_1)$$
$$= \frac{30}{100} * \frac{29}{99} = \frac{30\times29}{100\times99}$$

(b) $\mapsto$ 35-student tutorial

$$P(A_2B_2) = P(A_2)\cdot P(B_2|A_2)$$
$$= \frac{35}{100} \times \frac{34}{99}$$

$$P(A_3B_3) = P(A_2B_2)$$

$$P(A_1B_1) + P(A_2B_2) + P(A_3B_3) = \frac{30\times29 + 2\times35\times34}{100\times99}$$