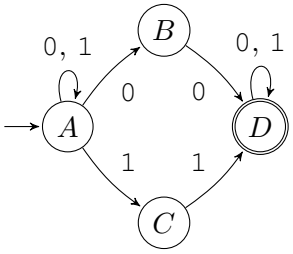
Week 3 Tutorial Session

1. (a) Write down a regular expression for the following NFA. For this problem, you do not have to go through the procedure described in class.

   (b) Convert the following NFA into a DFA.



2. Let L be any language. We say that two strings x and y are *indistinguishable by* L if for every string z, we have $xz \in L$ if and only if $yz \in L$.

   (a) For concreteness, consider $L_1 = \{x \in \{0,1\}^* \mid \text{the number of } 1\text{'s in } x \text{ is divisible by } 3\}$. Prove that `1` and `1111` are indistinguishable by L_1.

   (b) Continuing with (a), which strings are indistinguishable from the string `1` by L_1? The set of all such strings is the *equivalence class* of the string `1` and will be denoted by $[1]$.

   (c) Find a string s not in $[1]$. What is the equivalence class of s? (We will denote this equivalence class by $[s]$)

   (d) Can you find another string t not in $[1]$ or $[s]$? What is the equivalence class of t?

   (e) Can you find yet another string u not in these equivalence classes?

   (f) Design a DFA for the language L_1. How are states in your DFA related to the equivalence classes?

3. (a) Write down the definition of regular expressions over an alphabet Σ.

   (b) Given a string w, define w^R as the string w in reverse order. That is, if $w = w_1 w_2 \dots w_n$, then $w^R = w_n w_{n-1} \dots w_1$. For example, if $w =$ `live`, then $w^R =$ `evil`. Given a language L, define its reversal L^R as the set of strings in L in reverse. More precisely, $L^R = \{w^R \mid w \in L\}$. For example, if $L = \{\texttt{live}, \texttt{raw}, \texttt{level}\}$, then $L^R = \{\texttt{evil}, \texttt{war}, \texttt{level}\}$.

   If L is a regular language, prove that L^R as also regular.