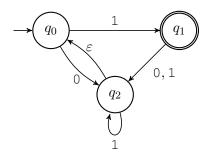Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (40 points) Give a DFA for the following languages, specified by a transition diagram. For each one of them, give a short and clear description of how the machine works. The alphabet is $\Sigma = \{0, 1\}$:

   (a) $L_1 = \{w \mid w$ has least three 1s or at least two 0s$\}$.

   (b) $L_2 = \{w \mid$ the sum of digits of w is not divisible by $3\}$.

   (c) L_3 is the language described by $(0+10^*1)^*$.

   (d) $L_4 = \{w \mid w$ contains the substring 10 an odd number of times$\}$.

(2) (10 points) Convert the following NFA to a DFA using the method described in class. Specify the DFA by its transition diagram. The alphabet is $\Sigma = \{0, 1\}$.

(3) (25 points) If w is a string, we say that a string x is an *initial part* of w if $w = xy$ for some string y. For example, b and bcd are both initial parts of $bcde$. Given a language L, define $L^I = \{x \mid x$ is an initial part of some $w \in L\}$. That is, L^I contains the initial parts of strings in L.

Prove that if L is a regular language, then so is L^I.

Hint: Regular language is defined *recursively*. If the desired result is true for simpler regular languages, can you show that it is also true for more complex regular languages?

(4) (25 points) In this problem you will design an NFA that checks if an input string correctly represents a simple mathematical expression, say to be used in a program that tests whether a primary school student has mastered those expressions. Examples of valid expressions are:

```
789
-1+234-0
```
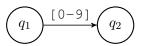
Examples of invalid expressions are:

```
012
12+-0
456-
```

For simplicity, assume the input string consist only of digits '0' through '9', and the symbols '+' (plus) and '−' (minus). The expression '012' is considered invalid because it should have been simplified to '12'.

  (a) (10 points) Give your definition of valid mathematical expressions. There is no single correct answer to this question. This question tests your ability to clearly state your definition. If you do not state your definition or if it is too difficult to understand, you will automatically get zero points for the remaining parts as well.

  (b) (10 points) Draw an NFA that accepts valid mathematical expressions (and nothing else).

  (c) (5 points) Explain how your NFA works in 3–6 sentences.

When drawing the transition diagram of your NFA, you can use the shorthand notation `[0-9]` to describe transitions labeled by all the digits '0', '1', ... '9'. An example of such a shorthand is

Similarly, you can use the notation `[1-9]` to describe the transitions labeled by all the non-zero digits.

This is a design problem, and part of your job is to figure out a way to distinguish among valid and invalid mathematical expressions. There may not be a single right answer. You must describe your assumptions and reasoning clearly in your solution. Solutions that only provide a picture of an NFA with no explanation will get no credit.