PROBABILITY MIDTERM REVIEW

1. 4-STEP METHOD
   EQUALLY LIKELY OUTCOMES
   PROBABILITY MODELS
   AXIOMS

2. CONDITIONAL PROBABILITIES
   TOTAL PROB. THEOREM
   BAYES' RULE

3. INDEPENDENCE OF EVENTS
   SEQUENTIAL & PARALLEL COMPONENTS
   CONDITIONAL INDEPENDENCE

RANDOM VARIABLES, PMF
Binomial(n,p), Geometric(p), Poisson(λ)
FUNCTIONS OF RVs

4. EXPECTED VALUE
   UTILITY
   JOINT PMF $\rightarrow$ MARGINAL PMFs
   LINEARITY OF EXPECTATION
   VARIANCE

$$\mathrm{Var}[X] = E[(X-\mu)^2] \qquad \mu = E[X]$$
$$= E[X^2] - \mu^2$$

5. CONDITIONAL PMFs
   CONDITIONAL EXPECTATION
   TOTAL EXPECTATION THEOREM
   INDEPENDENCE OF RVs
   LINEARITY OF VARIANCE FOR INDEPENDENT RVs
   FORMULAS FOR E/Var OF Binomial, Geometric, Poiss

EXAMPLE. ROULETTE.

INVEST \$100, BET 10% IN EVERY ROUND

X_t DOLLARS IN ROUND t

$$X_{t+1} = \begin{cases} 1.1 \cdot X_t & \text{IF I WIN} \quad 18/37 \\ 0.9 \cdot X_t & \text{IF I LOSE} \quad 19/37 \end{cases}$$

$$E[X_{t+1}] = E[X_{t+1} | W_{t+1}] \underline{P(W_{t+1})} + E[X_{t+1} | W_{t+1}^c] \underline{P(W_{t+1}^c)}$$

$$= 1.1\, X_t \cdot \frac{18}{37} + 0.9\, X_t \cdot \frac{19}{37}$$

$$= 0.997\ldots X_t$$

$$E[X_{10}] = 0.997^{10} \cdot X_0$$

INDEPENDENCE OF 3 RVS

2 RVS. $P(X=x, Y=y) = P(X=x)P(Y=y) \quad \forall x,y$

3 RVS. $P(X=x, Y=y, Z=z) = P(X=x)P(Y=y)P(Z=z) \quad \forall x,$

EXAMPLES WHERE

X, Y INDEPENDENT
X, Z INDEPENDENT
Y, Z INDEPENDENT

X, Y, Z DEPENDENT.

2 DICE

$$X = \begin{cases} 1 & \text{FIRST DIE IS A 1} \\ 0 & \text{IF NOT} \end{cases}$$

$$Y = \begin{cases} 1 & \text{SECOND DIE IS A 1} \\ 0 & \text{IF NOT} \end{cases}$$

$$Z = \begin{cases} 1 & \text{IF SUM IS A 7} \\ 0 & \text{IF NOT} \end{cases}$$

TO SHOW X, Y DEPENDENT ENOUGH TO GIVE ONE EXAMPLE

$$P(X=3, Y=5) \neq P(X=3)P(Y=5)$$

INDEPENDENT CHECK ALL x, y

WHY DOES LINEARITY OF EXPECTATION ALWAYS HOLD?

EX. TOSS 3-SIDED DIE D

X = INDICATOR FOR "$D=2$"
$Y = D^2$
} DEPENDENT

PROB	D	X		Y		X+Y
1/3	1	0	+	1	=	1
1/3	2	1	+	4	=	5
1/3	3	0	+	9	=	9

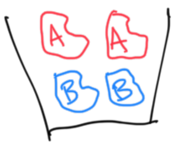
$E[X]=\frac{1}{3}$ + $E[Y]=\frac{14}{3}$

$E[X+Y]$ CAN BE CALCULATED IN TWO WAYS:

1. ADD THE PMFS AND AVERAGE
$\frac{1}{3}\cdot 1+\frac{1}{3}\cdot 5+\frac{1}{3}\cdot 9=\frac{15}{3}$
2. ADD THE AVERAGE
$\frac{1}{3}+\frac{14}{3}=\frac{15}{3}$

$$\boxed{E[X+Y]=E[X]+E[Y]}$$

QUIZ 4. ASSUME THERE ARE ONLY 2 PEOPLE, 4 SOCKS



S_A = "ALICE GETS BOTH SOCKS BACK"
S_B = "BOB "

$N = N_A + N_B$ $\qquad N_A = \begin{cases} 1 & \text{IF } S_A \text{ HAPPENS} \\ 0 & \text{IF NOT} \end{cases}$

N_A, N_B DEPENDENT $(P(N_B=1 \mid N_A=1)=1)$

$$\begin{aligned} E[N] &= E[N_A]+E[N_B] \\ &= P(S_A) + P(S_B) \\ &= 1/6 + 1/6 = 1/3 \end{aligned}$$

HS Q4b. $X_0 = 3$ DOLLARS

$$X_1 = \begin{cases} 2\cdot X_0 & \text{IF I WIN} \\ 0 & \text{IF I LOSE} \end{cases}$$

$$E[X_1] = 2X_0\cdot\frac{18}{37} + 0\cdot\frac{19}{37} = \frac{36}{37}X_0$$

$$E[X_2] = \left(\frac{36}{37}\right)^2 X_0$$

$$E[X_3] = \left(\frac{36}{37}\right)^3 X_0$$

PROFIT $P = X_3 - X_0$

$$E[P] = \left(\frac{36}{37}\right)^3 X_0 - X_0 \approx -0.08.$$