Alice is waiting for a bus that arrives at a rate of one per 10 minutes. Bob is equally likely to arrive at the same bus stop at any time within the next 10 minutes. What is the probability that Bob arrives at the bus stop before Alice boards her bus? (Assume the arrival times of the bus and Bob are independent.)

Solution: Let A be the time that Alice boards the bus and B be the arrival time of the bus. We model A and B as independent Exponential(1) and Uniform$(0, 1)$ random variables, respectively, where the time unit is 10 minutes. By the total probability theorem,

$$\mathrm{P}(A > B) = \int_{-\infty}^{\infty} \mathrm{P}(A > B \mid B = b) f_B(b) db = \int_0^1 \mathrm{P}(A > b) db = \int_0^1 e^{-b} db = 1 - 1/e \approx 0.632.$$