Ten people take off their left and right socks and put them in a common bag. Each person then takes out two random socks from the bag (without replacement). What is the expected number of people who recover both their socks?

Solution: Let X_i be an indicator random variable for the event that the i-th person recovers both his/her socks ($X_i = 1$ if this happens, $X_i = 0$ if it doesn't.) Then the number of people X who recover both their socks is

$$X = X_1 + X_2 + \cdots + X_{10}.$$

Even though the events $X_1 = 1, \ldots, X_{10} = 1$ are not independent, we can apply linearity of expectation to express $\mathrm{E}[X]$ as

$$\mathrm{E}[X] = \mathrm{E}[X_1] + \cdots + \mathrm{E}[X_{10}].$$

Since X_i is an indicator random variable, $\mathrm{E}[X_i] = \mathrm{P}(X_i = 1)$. Thus, the probability that person i recovers both his/her socks is

$$\mathrm{P}(X_i = 1) = 1/\tbinom{20}{2}$$

and so $\mathrm{E}[X] = 10/\binom{20}{2} = 1/19 \approx 0.053$.