

Jar  $A$  contains 3 black balls and 2 white balls. Jar  $B$  contains 1 black ball and 4 white balls. Alice draws a random ball from jar  $A$  and puts it in jar  $B$ . Bob then draws a random ball from jar  $B$ .

- (a) What is the probability that Bob's drawn ball is white?

**Solution:** Let  $A$  and  $B$  be the events that Alice and Bob draw a white ball respectively, and  $A^c$ ,  $B^c$  be the complementary events. We are given that  $P(A) = 2/5$  and  $P(A^c) = 3/5$ , and depending of the color of the moved ball we can calculate

$$P(B|A) = \frac{5}{6}, \quad P(B|A^c) = \frac{4}{6}.$$

By the total probability theorem,

$$P(B) = P(A) \cdot P(B|A) + P(A^c) \cdot P(B|A^c) = \frac{2}{5} \cdot \frac{5}{6} + \frac{3}{5} \cdot \frac{4}{6} = \frac{11}{15}.$$

- (b) Given the Bob drew a white ball, what is the probability that Alice moved a white ball?

**Solution:** By Bayes' rule,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(B)} = \frac{2/5 \cdot 5/6}{11/15} = \frac{5}{11}.$$