Alice, Bob, and Charlie hold a lucky draw for two tickets to a concert with the following odds:

- The probability that Alice gets one of the tickets is 50%.
- The probability that Bob gets one of the tickets is 70%.

What is the probability that Alice and Bob both get tickets?

Solution: The sample space consists of the three outcomes $\{ab, ac, bc\}$, where ab represents Alice and Bob getting the tickets, and so on. Denote their probabilities by p_{ab}, p_{ac}, and p_{bc}. The event "Alice gets one of the tickets" is $\{ab, ac\}$ so $p_{ab} + p_{ac} = 0.5$. Similarly $p_{ab} + p_{bc} = 0.7$. Since the probabilities must add up to one,

$$p_{ab} = (p_{ab} + p_{ac}) + (p_{ab} + p_{bc}) - 1 = 0.5 + 0.7 - 1 = 0.2.$$

Alternative solution: Let A, B, and C be the events "Alice gets a ticket" and so on. By the axioms the complementary events have probabilities $\mathrm{P}(A^c) = 0.5$ and $\mathrm{P}(B^c) = 0.3$. The events A^c, B^c, and C^c partition the sample space (they are disjoint and exactly one of the three must occur) so their probabilities must add up to one. Therefore $\mathrm{P}(C^c) = 1 - 0.5 - 0.3 = 0.2$. The event C^c happens exactly when Alice and Bob both get tickets, so the desired probability is 20%.