ENGG 2430A: Probability and Statistics for Engineers
The Chinese University of Hong Kong, Spring 2020 Practice Midterm 1

Each question is worth ten points. To receive full credit for your answer, you must clearly describe the sample space, the event of interest, and explain your calculations.

1. 3 red balls and 3 blue balls are randomly arranged on a line. Let X be the position of the first blue ball. (E.g. for the arrangement `RBRBBR`, $X = 2$.) Find the probability mass function of X.

2. Half the students know the answer to a true-false question. The other half guesses at random. I ask a random student and his answer is correct. What is the probability he knows the answer?

3. Toss a coin 4 times. Let X, Y and Z be the number of heads among the first two, middle two, and last two tosses, respectively. Are X and Z independent given that $Y \neq 1$? Justify carefully.

4. The average lifetime of a lightbulb is 10 months. You install 10 lightbulbs today. What is the probability that at least one of them fails within a month? Assume their failures are independent.

5. Eight people's hats are mixed up and randomly redistributed. What is the expected number of pairs that exchanged hats (Alice got Bob's and Bob got Alice's)?