On the Access Pricing and Network Scaling Issues of Wireless Mesh Networks

Ray K. Lam Dah-Ming Chiu John C.S. Lui*

***Abstract*— Distributed wireless mesh network technology is ready for public deployment in the near future. Yet without an incentive system, one should not assume private, self-interested wireless nodes would participate in such a public network and cooperate in the packet forwarding service. This paper studies the use of pricing as an incentive mechanism to stimulate participation and collaboration in public wireless mesh networks. Our focus is on the "economic behavior" of the network nodes—the pricing and purchasing strategies of the access point, wireless relaying nodes, and clients. We use a "game theoretic approach" to analyze their interactions from one-hop to multi-hop networks and when the network has an unlimited or limited channel capacity. The important results we show are that the access point and relaying wireless nodes will adopt a simple, yet optimal, fixed-rate pricing strategy in a multi-hop network with an unlimited capacity. Yet, the access price grows quickly with the hop distance between a client and the access point, which may limit the "scalability" of the wireless mesh network. In the case that the network has a limited capacity, the optimal strategy for the access point is to vary the access charge and may even interrupt service to connecting clients. To this end, we focus on the access point adopting a non-self-enforcing but more practical "fixed-rate, non-interrupted service" model, and propose an algorithm based on the Markovian decision theory to devise the optimal pricing strategy. Results show that the scalability of a network with a limited capacity is upper bound by one with an unlimited capacity. We believe this work will shed light on the deployment and pricing issues of distributed public wireless mesh networks.**

Keywords: Wireless networks, economics, game theory, Markov decision process.

I. Introduction

In recent years, we have seen a growing interest of wireless mesh network technology, and simultaneously the growing popularity of wireless network devices, at homes, offices and public places such as cafes, malls and hotels. The two induce a vision when wireless mesh network technology is deployed in the public, we would have nearly ubiquitous wireless coverage in large urban areas, provided that a vast number of private wireless access points and devices participate in such a mesh network. Another justification of using the mesh technology is to bring Internet access to developing areas where wired network infrastructure is not readily available.

Yet an important question left unanswered is why private access points and wireless nodes would participate in a public mesh network and act in a cooperative manner. Connectivity in a mesh network relies on nodes forwarding packets for each other, but relaying packets incurs costs to a node, in terms of reduced bandwidth, energy consumption, potential security risks, etc. In community, experimental or proprietary mesh networks, cooperation can be assumed, but in order that wireless mesh network goes beyond community borders and provides ubiquitous wireless coverage to the general public, we have to take note that nodes in the network will be private, self-interested, or economically rational. Without incentives, one should not assume these nodes to cooperate.

In this work, we study the use of pricing as a mechanism to stimulate participation and collaboration in a public mesh network. As the objective of most nodes would be to access the Internet, we take "*Internet access*" as a service, and hence access points are the service sellers. Any downstream wireless nodes may purchase this service, for her own consumption, or for reselling it to nodes further downstream. Transactions involved must be on a per-access basis, using technology such as the PayWord micro-payment scheme [1], [2], which minimizes the transaction overhead. Monthly prepayment scheme (such as those implemented in proprietary wireless mesh access networks) or the like is impossible as nodes here concerned are not reliable to provide consistent service in the long run. By this access provision business, participating nodes generate revenue to compensate their costs for packet forwarding. We investigate the pricing and purchasing behavior of different nodes in the network. We seek to answer these questions:

- How will the access point and different wireless relaying nodes set their prices for the service?
- Will their optimal pricing schemes be complicated, such as the access point charging a floating rate with time, which may discourage clients for the service?
- How many clients can afford the price and eventually receive the Internet service?
- Do we need third-party supervision to enforce the price?
- Is it economically scalable to extend the network in a multi-hop fashion? Will the price charged to a distant client be too high after the relaying nodes add in their costs and desired gains?

We believe answers to these questions will shed light on the deployment of public wireless mesh networks.

Our analysis adopts a game theoretic approach to find out the strategies that the access points, relaying wireless nodes, and clients will play throughout the bargaining process at equilibrium. We focus on mesh networks in which there is a single access point having the Internet connectivity, and every wireless client has a single path toward this access point.

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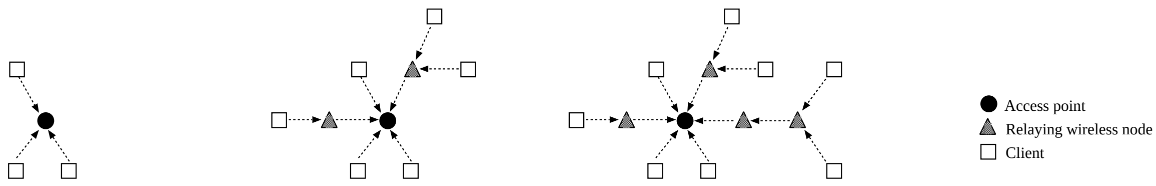


(a) One-hop wireless network (b) Two-hop wireless network (c) Multi-hop wireless network

Fig. 1. Various wireless mesh networks analyzed in this paper

Figure 1 shows three examples of such a tree-like network. We differentiate two cases in this setting: (1) the wireless network and the access point's wired uplink to the Internet have an unlimited capacity (or the capacity is sufficiently large to satisfy all demands); (2) the network has a limited capacity. In each case, we first look at a one-hop network depicted in Figure 1(a), in which all clients can reach the access point directly; then we extend it to the multi-hop case as in Figure 1(c), in which clients have to route through numerous relaying wireless nodes, or resellers, to the access point, in order to receive the Internet access service. Note that the one-hop case and two-hop case (Figure 1(b)) under the unlimited capacity assumption are first studied in the seminal in [3]. Studying pricing under the unlimited capacity assumption is worthwhile, as it provides asymptotic results as the wired and wireless network capacity go abundant, which can be foreseen due to technology maturity. The limited capacity model offers a more realistic investigation, and we expect the access point to play a rather different strategy when she can only sell her service to a limited number of clients. Adopting a tree-like network model simplifies the problem and provides us the basic pricing structure in wireless mesh networks. As discussed later, results in tree-like networks will serve as building blocks for pricing structure in networks with a general topology.

The contributions of this work are summarized as follows. First, we generalize the model in [3] and show that it is only a special case when the network has an unlimited capacity, or equivalently, has an adequate supply of bandwidth to meet all demands from clients. The elegant results in the unlimited capacity model—the access point and resellers charging a "*fixed rate*" at all time—no longer apply in the limited capacity case. Secondly, we extend the two-hop case of the unlimited capacity model in [3] to the multi-hop case, and conclude that sparseness of nodes in a wireless mesh network results in low economic scalability of the network. Third, in view of the fixed-rate pricing strategy being non-optimal when the network has a limited capacity, we propose a more practical charging policy, the '*fixed-rate, non-interrupted service*", for wireless Internet access. Under this policy, we use the policy-iteration method from the Markovian decision theory to devise the optimal pricing strategy of the access point. The algorithm is made applicable to both the one-hop case and the multi-hop case of the limited capacity model.

The balance of this paper is as follows. In Section II we discuss background and related results in [3]. In Section III we study the unlimited capacity model with explicit client's utility distributions in its one-hop case, then extend it to the multi-hop case by proposing an equilibrium strategy profile and analyze its scaling issue. In Section IV we investigate the limited capacity model, showing the previous equilibrium no longer holds, then we present the fixed-rate, non-interrupted service model and devise the optimal pricing strategy of the access point using the Markovian decision theory. We finish the section with an analysis of the multi-hop case and some observations on the network scaling issue of the limited capacity model. Section V concludes.

II. **Related Work and Background**

Pricing in computer networks has been receiving much attention from the community recently. Research efforts have first been made on pricing in wired networks, and then in wireless hotspot networks, wireless ad-hoc networks and wireless mesh networks. Our work differs from existing works mainly in two ways. First, our paper focuses on multi-hop wireless mesh networks and investigates whether general pricing mechanisms are effective to provide incentives to build such networks in a scalable fashion. Second, as to answer the first question, we model the utility of all wireless nodes and allow the theoretically largest action space in the pricing game. Each selfish node will then maximize her own utility in the game, and the pricing equilibrium resulted is studied to give insights to the practical design of public wireless mesh networks which exploit pricing as an incentive system.

Pricing in wired networks has been studied by MacKie-Mason *et al.* [4] and Kelly *et al.* [5]. They investigate the use of pricing as a method to regulate network traffic in view of congestion and promote network efficiency. In [6], Paschalidis and Liu further prove that in a network with many small users, static pricing is asymptotically optimal. In [7] Campos-Náñez and Patek present, when the assumption of many small users does not hold, a computational procedure for optimal static pricing in response to real-time client arrivals and departures. In [8] Viterbo and Chiasserini study a similar issue but places the problem scenario in a wireless network. In [9], the authors present a distributed pricing scheme to eliminate anomaly when multiple overlays interact with each other.

A number of researchers have investigated pricing in wireless networks as a mechanism to promote participation and cooperation in packet forwarding among wireless clients, using different modeling approaches and under different assumptions. Friedman and Parkes study a strategy-proof VCG fixed-rate pricing scheme for WiFi and wireless ad-hoc networks [10]. Chen *et al.* propose pricing mechanisms for different multi-hop network structures based on a demand-and-supply market model [11]. Anderegg and Eidenbenz develop a VCG routing protocol which achieves truthfulness from wireless clients and cost efficiency by assuming a selfish relaying node will forward a packet if her cost of forwarding is covered [12]. Musacchio and Walrand shows that fixed-rate pricing is optimal in a WiFi network when users have a "web browsing utility", under the assumption that the network has an unlimited capacity [3]. Zhong *et al.* propose a cooperation-optimal routing and forwarding protocol for wireless ad-hoc networks, which integrates VCG mechanism and cryptographic technique [13]. In [14], Wang *et al.* present a multicast routing protocol for wireless networks without using VCG mechanism. The protocol guarantees truthfulness from clients. In [15], [16], the authors present an incentive mechanism and service differentiation policy so as to promote contribution in P2P networks.

The seminal work by Musacchio and Walrand [3] presents the economic behavior of wireless nodes under a specific network topology. In particular, they study "one-hop" and "two-hop" wireless networks using a game theoretic approach, and prove that "*fixed-rate pricing*" is optimal to the access point, given that clients have the so-called "web browsing" utility function. Web browsing utility function models, for a client browsing the web, her utility of having Internet access—the utility grows proportionally with the time she gains access initially, and saturates when she no longer intends to browse.[1] Note that the analysis adopted and the results proven are only valid under a strong assumption: the network has an *unlimited* capacity, i.e. the channel capacity of the wireless network is unlimited and the access point has an unlimited uplink bandwidth to the Internet, or the access point provides no bandwidth guarantee to clients, while clients value the connection service without considering the available bandwidth. This assumption allows the access point to admit infinitely many clients; the admission of one client has no influence on the admission of others. Thus, the access point's total profit can be maximized by separately maximizing her gain in each interaction with a client. In the one-hop case, a two-player game between the access point and a single client abstracts all details of the aggregated system; while in the two-hop case, a three-player game among the access point, a single relaying node, and a single client will do. Our work relaxes the unlimited capacity assumption and shows that fixed-rate pricing is no longer optimal to the access point. The model adopted by [3] is hereafter termed unlimited capacity model. In the following, we first present the related results, which serve as the basis of our work.

A. Unlimited Capacity Model—One-hop Case

The one-hop case of the unlimited capacity model describes a wireless network where all clients can reach the access

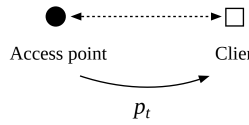


Fig. 2. Game modeling of the one-hop case with a slot price p_t charged by the access point

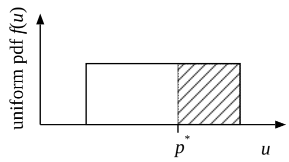


Fig. 3. A uniform distribution of client's per-slot utility U, with shaded area representing $P(U \geq p^*)$, the expected proportion of connecting clients when the access point charges at price p^*.

point directly (i.e., without the need of packet forwarding by other nodes). The dynamics among the access point and the numerous clients is captured using a two-player game between the access point and a single client as shown in Figure 2. Time is divided into discrete slots. At the beginning of each time slot t, the client requests for connection service over the slot and the access point replies with a slot price p_t. The client chooses to accept the price and connect to the access point, or to reject and leave. The game ends once the client rejects a slot price, and the number of time slots the client connects is denoted by T. The client has a web browsing utility function:

$$F(T, \tau) = U \cdot \min(T, \tau),$$

where τ is a *discrete random variable* representing the number of time slots the client intends to connect and browse the web, and U is a *continuous random variable* representing the client's utility of gaining Internet access in one time slot. The client knows her values of U and τ, while the access point's prior knowledge of them includes only their probability distributions, obtained for example from market survey. Figure 3 depicts one possible distribution of client's per-slot utility U—a uniform distribution. At the end of the game, the client has a net payoff of $F(T, \tau) - \sum_{t=1}^{T} p_t$, while the access point has a profit of $\sum_{t=1}^{T} p_t$. Authors in [3] prove that the following strategy is a perfect Bayesian equilibrium (PBE) [17]:

- The client connects or remains connected in slot t iff $t \leq \tau$ and $p_t \leq U$;
- The access point charges a non-decreasing price sequence $\{p_t\}$ such that $p_t \in \arg\max_p pP(U \geq p)$.

There are three points to be noted here. First, the client's strategy is named the "*myopic strategy*", for its sole dependence on the immediate slot price. Second, it is often the case that the access point charges a "constant", or fixed price sequence, since the expression $pP(U \geq p)$ is maximized by a single price p^* for most distributions of U, and the price p^* does not vary over time slots. Third, the quantity $P(U \geq p^*)$, as shaded in Figure 3 for a uniform distribution of U, has a physical meaning of the expected proportion of clients who are willing to pay and connect to the access point at the PBE.

[1] In [3] the authors also study another type of utility function called the "file transfer" utility function. It models the case in which a client is downloading a file, who must remain connected before the download is finished in order to earn any utility. The function is like a step function. As the authors in [3] point out that such utility is uncommon today since software for downloading or sharing large files often provides the "resume" function for broken download, we will not further the analysis with this utility in our work.

This is because each instance of price negotiation between the access point and a client at the first time slot can be taken as a Bernoulli trial with a probability of $P(U \geq p^*)$ that the negotiation is a success. With n independent negotiations with n clients, the number of successes is a binomial random variable with an expected value of $nP(U \geq p^*)$. Hence the expected proportion of clients willing to pay and connect the access point is $P(U \geq p^*)$.

B. Unlimited Capacity Model—Two-hop Case

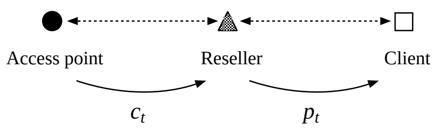


Fig. 4. Game modeling of the two-hop case with a slot price c_t charged by the access point, and a slot price p_t charged by the reseller

The two-hop case describes the situation when a client is incapable of reaching the access point directly, but has to route her traffic through an intermediate wireless node, referred as the reseller. The game now involves three players, with the additional reseller, as shown in Figure 4. At the beginning of each time slot t, the client requests service from the reseller. The reseller in turn requests service from the access point, who replies her with a slot price c_t. Based on c_t, the reseller decides how to charge and sends a slot price p_t to the client. The client chooses to accept or reject the price. If the client accepts p_t, the reseller replies "accept" to the access point; and vice versa. When game ends, resulted from the first rejection of a slot price by the client, the net payoff of the client is $F(T,\tau) - \sum_{t=1}^{T} p_t$, while the reseller and the access point have profit of $\sum_{t=1}^{T}(p_t - c_t)$ and $\sum_{t=1}^{T} c_t$ respectively. Authors in [3] prove that the following strategy profile is a PBE:

- The client follows the myopic strategy, connecting iff $t \leq \tau$ and $p_t \leq U$;
- The reseller picks a price mark-up function $p^*(c)$ that satisfies the properties:

$$
\begin{aligned}
p^*(c) &\in \arg\max_p (p-c)P(U \geq p) \\
p^*(c') &\geq p^*(c) \qquad \forall c' > c
\end{aligned}
$$

  and charges the price $p_t = p^*(c_t)$ in slot t;
- The access point charges a non-decreasing price sequence $\{c_t\}$ such that $c_t \in \arg\max_c cP(U \geq p^*(c))$.

As in the one-hop case, it is common for the access point and the reseller to adopt a fixed-price strategy, since most distributions of U yield single maximizers of $(p-c)P(U \geq p)$ and $cP(U \geq p^*(c))$ respectively.

The most important result of [3] is the proof of the natural selection of the fixed-rate pricing strategy by the access point and the reseller, without the need of contract enforcement. Fixed-rate pricing is appealing to customers for its simple charging scheme; while the exclusion of contract enforcement allows the service mechanism to be on a pure peer-to-peer basis and hence be scalable. However, as we are going to show in Section IV, this result only applies to the following special situations:

- The wireless network channel and the access point's uplink have an unlimited capacity, or have a sufficient capacity to meet all demands;
- The network has a limited capacity, but the access point does not provide bandwidth guarantee to clients; while clients' valuations of the service are independent to its quality.

Clearly, the first condition is not always true while the second condition may not be realistic. For networks where the above conditions do not hold, the pricing and purchasing strategies remain to be investigated.

Remark: It is shown in [3] that a one-hop or two-hop wireless mesh network in which the network has an unlimited capacity, clients will be charged at a fixed rate by the access point or relaying wireless nodes.

III. Extensions to the Unlimited Capacity Model

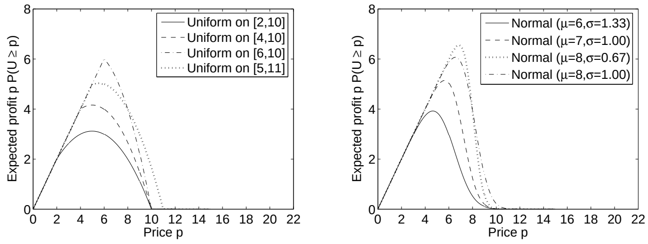
The game theoretic modeling in [3] provides a useful methodology to analyze the pricing dynamics in a wireless mesh network; one in which either the network has an unlimited capacity, or clients do not differentiate services of different bandwidths but only require an Internet access. In this section, we provide a methodology which offers a more comprehensive analysis. We first examine the game PBE in its one-hop case with various probability distributions of client's per-slot utility U, followed by a natural extension of the analysis into the multi-hop case. We then discuss some important network scaling issues in the economic perspective of such a network.

A. Optimal Pricing for the One-hop Case under Various Utility Distributions

In [3], authors provide generalized perfect Bayesian equilibria, applicable for any arbitrary distribution of client's per-slot utility U, for both the one-hop and two-hop case. It would be helpful to analyze the one-hop case PBE with some sample utility distributions before moving on to the more sophisticated multi-hop case. In the following, we study the one-hop case PBE with the uniform utility distribution, for its mathematical tractability to obtain closed-form results; and the normal utility distribution[2], for its realistic representation of a real-world market. In particular, we are interested to obtain p^*, the optimal price per slot, with which the access point maximizes $pP(U \geq p)$; and also the quantity $P(U \geq p^*)$, which physically represents the proportion of clients who are willing to pay the price p^* so as to obtain the wireless connection.

Consider the case that the client's per-slot utility U has a uniform distribution on the interval $[a, b]$ with $a \leq b$. Any price lower than a would be accepted by a client, so the access point can set a price at a which outperforms all such prices. Any

[2]When we use the normal utility distribution, the "tail" of the probability density function which falls in the negative region is not truncated. A client having a negative utility physically represents, for example, a user who is not interested in browsing the web, and the access service implies a cost to her, such as the cost on battery usage.



(a) U uniformly distributed (b) U normally distributed

Fig. 5. Expected profit per slot per client ($pP(U \geq p)$) vs. price p

Distribution	p^*	$P(U \geq p^*)$	$pP(U \geq p^*)$
Uniform on $[2, 10]$	5.00	0.63	3.13
Uniform on $[4, 10]$	5.00	0.83	4.17
Uniform on $[6, 10]$	6.00	1.00	6.00
Uniform on $[5, 11]$	5.50	0.92	5.04
Normal ($\mu = 6$,$\sigma = 1.33$)	4.66	0.84	3.91
Normal ($\mu = 7$,$\sigma = 1.00$)	5.65	0.91	5.15
Normal ($\mu = 8$,$\sigma = 0.67$)	6.86	0.96	6.56
Normal ($\mu = 8$,$\sigma = 1.00$)	6.56	0.93	6.07

TABLE I

THE PBE PRICING FOR THE ONE-HOP NETWORK

price higher than b would be rejected by a client, hence a price higher than b yields zero expected payoff. Thus we are sure that the optimal price for the access point lies on the interval $[a, b]$. With this assumption, we may write:

$$pP(U \geq p) = p\left(\frac{b-p}{b-a}\right). \tag{1}$$

Differentiating Eq. (1) with p and equate it to zero yield the unique PBE price p^* set by the access point:

$$p^* = \begin{cases} b/2 & \text{if } a \leq b/2 \\ a & \text{otherwise.} \end{cases}$$

The price sequence $\{p_t\}$ is hence a *fixed* sequence with $p_t = p^*$ for all time slot t. At the PBE price p^*, the proportion of connecting clients is:

$$P(U \geq p^*) = \begin{cases} \frac{b}{2(b-a)} & \text{if } a \leq b/2 \\ 1 & \text{otherwise.} \end{cases}$$

The closed-form results can be confirmed with the numerical examples in Figure 5(a) and Table III-A, which test four uniform distributions of U on the interval $[2, 10]$, $[4, 10]$, $[6, 10]$ and $[5, 11]$ respectively. Figure 5(a) plots the function $pP(U \geq p)$, or the expected profit per slot for the access point, against p for the four different distributions. It can be observed that each curve is composed of three parts: on the interval $[0, a]$, the function grows linearly with p, since a price below a will definitely be accepted by a client; on $[a, b]$, the curve has a single maximization point, at which p^* is located; on $[b, \infty]$, the function has zero value since no client will be willing to pay a price higher than b. Except for the third distribution, uniform on $[6, 10]$, all distributions have $a \leq b/2$ and p^* at $b/2$, which are classified as normal cases. The remaining distribution has p^* at a, contributing the boundary case, as p^* is at the boundary of $[a, b]$.

When the utility U has a normal distribution, closed-form results are no longer tractable and one has to resort to numerical analysis. Four different normal distributions are used and the results are illustrated in Figure 5(b) and Table III-A. The curves again possess the three-part characteristic as discussed above, but there are no discrete boundaries between the composing parts due to the smoothness of the cumulative density function of normal distribution. Note that the maximizing price p^* is still unique in this case.

B. Optimal Pricing for Multi-hop Wireless Network

We now extend the unlimited capacity analysis into the multi-hop case, which is derived naturally from the two-hop case in [3]. The multi-hop case describes pricing dynamics in a multi-hop wireless network where a client is at an arbitrary number of hops away from the access point. We first define the model and some notations, then propose a game PBE, and follow it by examples under various distributions of client's per-slot utility. Results in this section also contribute to the solution to the multi-hop case under the limited capacity model later in Section IV-B.

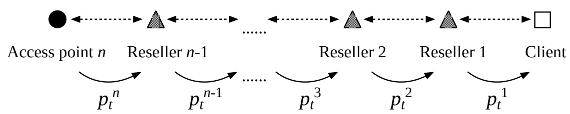


Fig. 6. Game modeling of the multi-hop case

The multi-hop case allows clients to be arbitrarily n-hop away from the access point. Due to the unlimited capacity assumption, we can abstract the aggregated system to a game involving all nodes on the path from a client to the access point. The game involves $n+1$ players: the access point, $n-1$ resellers and the client. as shown in Figure 6. The resellers are indexed from the client side to the access point side by 1 to $n-1$, while the access point is indexed by n. Procedures for price negotiation are analogous to that in the two-hop case: at each time slot t, access point n charges reseller $n-1$ a price p_t^n, who in turn charges reseller $n-2$ a price p_t^{n-1} and so on; in the end, the client receives a price p_t^1 from reseller 1. The net payoff of the client is $F(T, \tau) - \sum_{t=1}^{T} p_t^1$ for a usage of T time slots. The net payoff for reseller i is $\sum_{t=1}^{T}(p_t^i - p_t^{i+1})$, for $i = 1, \ldots, n-1$, and for the access point, the payoff is $\sum_{t=1}^{T} p_t^n$.

To determine the optimal pricing and a perfect Bayesian equilibrium for the unlimited capacity, multi-hop wireless network, we propose the **Strategy Profile 1**.

It can be proven that the above strategy profile is indeed a PBE. The proof follows naturally from the proof of the two-hop case PBE in [3]. The complete proof is obtainable in our technical report [18].[3]

To see how the PBE of the game in the multi-hop case works, let us consider examples in which client's per-slot

[3]The proof is also included in the appendix of this manuscript for the completeness of presentation only.

Strategy Profile 1 For Multi-hop Wireless Mesh Network

(1) The client follows the myopic strategy, connecting iff $t \le \tau$ and $p_t^1 \le U$;
(2) Reseller i, for all $i \in \{1, \ldots, n-1\}$, picks a price mark-up function $p^{i*}(p^{i+1})$ that satisfies the properties:

$$p^{i*}(p^{i+1}) \in \arg\max_{p^i}[(p^i - p^{i+1})P(U \ge m^i(p^i))] \quad (2)$$

$$p^{i*}(p^{i+1\prime}) \ge p^{i*}(p^{i+1}) \qquad \forall p^{i+1\prime} > p^{i+1} \quad (3)$$

and charges the price $p_t^i = p^{i*}(p_t^{i+1})$ in time slot t;
(3) Access point n charges a non-decreasing price sequence $\{p_t^n\}$ with $p_t^n \in \arg\max_{p^n}[p^n P(U \ge m^n(p^n))]$, where the function $m^i(p^i)$ is defined for all $i \in \{1, \ldots, n\}$ as follows:

$$m^i(p^i) \triangleq \begin{cases} p^{1*}(p^{2*}(\ldots(p^{i-1*}(p^i))\ldots)) & \forall i \in \{2, \ldots, n\} \\ p^1 & i = 1. \end{cases} \quad (4)$$

The function $m^i(p^i)$ represents the price received by the client after the price p^i set by node i is marked up by all its downstream resellers.

utility U is uniformly and normally distributed. At the PBE of each of the following examples, the expression $p^n P(U \ge m^n(p^n))$ is maximized by a single p^n and $(p^i - p^{i+1})P(U \ge m^i(p^i))$ is maximized by a single p^i, for all $i \in \{1, \ldots, n-1\}$. For convenience, we use the notation p^{i*} to denote the maximizing price of node i, for all $i \in \{1, \ldots, n\}$. At the PBE, the price sequence $\{p_t^i\}$ of each node i is fixed at $p_t^i = p^{i*}$ for all time slot t. Generally speaking, evaluation of p^{i*} for each node i involves a recursive iteration process. In the special case that U is uniformly distributed, one can obtain a closed-form solution of p^{i*} for each node i.

Let us first consider the case that U is a uniform distribution.

Theorem *1:* In the multi-hop case of the unlimited capacity model, when U is uniformly distributed on the interval $[a, b]$ with $a \le b$, access point n will charge reseller $n-1$ at $p^{n*} = b/2$ at the PBE, independent of the path length n toward the client, given that the "normal case" condition $(2^{n-1})a - (2^{n-1}-1)b \le b/2$ is satisfied.

Proof: Reseller 1's optimal price p^{1*} is to be paid by the client. Hence, p^{1*} must be no less than a as in the one-hop case. Also, a price of b yields zero payoff as will any price greater than b. Thus, we assume p^{1*} lies on the interval $[a, b]$. With this assumption, we may write:

$$(p^1 - p^2)P(U \ge m^1(p^1)) = (p^1 - p^2)(\frac{b - p^1}{b - a}). \quad (5)$$

Differentiating the above equation with p^1 and equating the result to zero yield: $p^{1*} = (p^2 + b)/2$. If $(p^2 + b)/2 > b$, it means that reseller 2 charges reseller 1 with price $p^2 > b$. We assume reseller 1 picks the price $p^1 = b$ in this case as the client will not connect anyway. If $(p^2 + b)/2 < a$, it happens that Eq. (5) has no stationary point on the interval $[a, b]$, and hence the maximum point is at the boundary and must be at a. We modify the optimal price of reseller 1 as follows:

$$p^{1*} = \begin{cases} a & \text{if } (p^2 + b)/2 < a \\ (p^2 + b)/2 & \text{if } (p^2 + b)/2 \in [a, b] \\ b & \text{otherwise.} \end{cases}$$

Now we look at reseller 2. Reseller 1 will charge at a if the condition $(p^2 + b)/2 < a$ is satisfied. This condition can be re-written as $p^2 < 2a - b$, and hence we see that reseller 2 should not pick any price lower than $2a - b$, as reseller 1 will charge the client at price a anyway. In a similar fashion as we analyze reseller 1, we assume reseller 2's optimal price $p^{2*} \in [2a - b, b]$. With this assumption, reseller 1 will price up p^{2*} by $(p^{2*} + b)/2$. Hence we write:

$$(p^2 - p^3)P(U \ge m^2(p^2)) = (p^2 - p^3)(\frac{b - (p^2 + b)/2}{b - a}).$$

Adopting the previous optimization technique, we have $p^{2*} = (p^3 + b)/2$, and as we do to p^{1*}, we need to modify the optimal price of reseller 2 to:

$$p^{2*} = \begin{cases} 2a - b & \text{if } (p^3 + b)/2 < 2a - b \\ (p^3 + b)/2 & \text{if } (p^3 + b)/2 \in [2a - b, b] \\ b & \text{otherwise.} \end{cases}$$

We can iterate this process upstream through the numerous resellers. If, for node i, every downstream reseller j adopts the price mark-up strategy $p^{j*} = (p^{j+1} + b)/2$, we can express:

$$p^{1*} = m^i(p^i) = \frac{(2^{i-1} - 1)b + p^i}{2^{i-1}} \qquad \forall i \in \{1, \ldots, n\}.$$

Note that this result also holds for access point n. One can find that for every reseller i, $i \in \{1, \ldots, n-1\}$. In particular, if $(p^{i+1} + b)/2 < (2^{i-1})a - (2^{i-1} - 1)b$, then $p^{i*} = (2^{i-1})a - (2^{i-1} - 1)b$. If $(p^{i+1} + b)/2 \in [(2^{i-1})a - (2^{i-1} - 1)b, b]$, then $p^{i*} = (p^{i+1} + b)/2$. For other cases, $p^{i*} = b$.

Finally, access point n should charge reseller $n-1$ at least $(2^{n-1})a - (2^{n-1} - 1)b$. Assuming $p^{n*} \in [(2^{n-1})a - (2^{n-1} - 1)b, b]$, we may write:

$$p^n P(U \ge m^n(p^n)) = p^n(\frac{b - ((2^{n-1} - 1)b + p^n)/2^{n-1}}{b - a}),$$

giving

$$p^{n*} = \begin{cases} b/2 & \text{if } (2^{n-1})a - (2^{n-1} - 1)b \le b/2 \\ (2^{n-1})a - (2^{n-1} - 1)b & \text{otherwise.} \end{cases} \quad (6)$$

Due to the choice of p^{n*} by access point n, each reseller i, for all $i \in \{1, \ldots, n-1\}$, marks up upstream price by $p^{i*} = (p^{i+1*} + b)/2$, and hence we have the following explicit form of the PBE price of reseller i, for all $i \in \{1, \ldots, n-1\}$:

$$p^{i*} = \begin{cases} (2^{n-i+1} - 1)b/2^{n-i+1} & \text{if } (2^{n-1})a - (2^{n-1} - 1)b \le b/2 \\ (2^{i-1})a - (2^{i-1} - 1)b & \text{otherwise.} \end{cases} \quad (7)$$

A case satisfying the condition $(2^{n-1})a - (2^{n-1} - 1)b \le b/2$ is classified as a *normal case*; else a *boundary case*. The boundary case corresponds to one in which reseller 1 charges the client a fixed price $p^{1*} = a$, at the boundary of the interval $[a, b]$. It can be observed from Eq. (6) that, in normal cases, access point n always charges a fixed price $b/2$, independent of n, which is the path length toward the client. ∎

The above results can be confirmed with numerical examples depicted in Figure 7 and 8. We define

$$w^i(p^i) \triangleq \begin{cases} p^i P(U \ge m^i(p^i)) & i = n \\ (p^i - p^{i+1*})P(U \ge m^i(p^i)) & \forall i \in \{1, .., n-1\}. \end{cases}$$

The function $w^i(p^i)$ is the one to be maximized by node i in the PBE, with her choice of the price she charges her downstream hop. Furthermore, $w^i(p^i)$ includes the PBE price p^{i+1^*} and the PBE aggregated price mark-up function $m^i(p^i)$ in its expression. These curves thus show how a node i picks the price p^{i^*} to maximize $w^i(p^i)$, assuming all other nodes play their respective PBE strategies. Figure 7 plots $w^i(p^i)$ against p^i for each node i in the four-hop case. In particular, Figure 7(a) shows a normal case with U uniformly distributed on $[0, 10]$; Figure 7(b) shows a boundary case with U uniformly distributed on $[9.5, 10]$. Figure 8(a) shows the PBE price p^{i^*} of each node i in the 2-hop, 4-hop and 8-hop case with U uniformly distributed on $[0, 10]$. The curves coincide with each other as suggested by Eq. (6) and (7). For a client located n-hop away from the access point, the probability that she accepts the price p^{1^*} and connects is

$$P(U \geq p^{1^*}) = \begin{cases} \frac{1}{2^n}(\frac{b}{b-a}) & \text{if } 2^{n-1}(a-b)+b \leq b/2 \\ 1 & \text{otherwise.} \end{cases} \tag{8}$$

Notice that in a boundary case, the client always accepts the price. Another quantity of interest is the expected payoff of each node. The expected payoff of each node i can be compared through $w^i(p^{i^*})$, which has a physical meaning of node i's expected profit at the first time slot. It can be expressed as:

$$\begin{aligned} w^n(p^{n*}) = p^{n*}P(U \geq m^n(p^{n*})) \;&=\; p^{n*}P(U \geq p^{1^*}) \\ &= \begin{cases} \frac{1}{2^{n+1}}(\frac{b^2}{b-a}) & \text{if } 2^{n-1}(a-b)+b \leq b/2 \\ (2^{n-1})(a-b)+b & \text{otherwise,} \end{cases} \end{aligned}$$

and similarly, for all $i \in \{1, \ldots, n-1\}$:

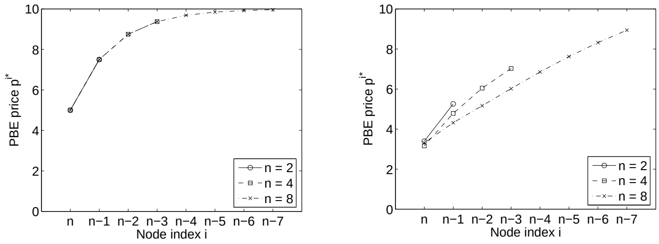
$$\begin{aligned} w^i(p^{i^*}) &= (p^{i^*} - p^{i+1^*})P(U \geq m^i(p^{i^*})) \\ &= \begin{cases} \frac{1}{2^{2n-i+1}}(\frac{b^2}{b-a}) & \text{if } 2^{n-1}(a-b)+b \leq b/2 \\ 2^{i-1}(b-a) & \text{otherwise.} \end{cases} \end{aligned}$$

Figure 9(a) plots $w^i(p^{i^*})$ of access point n and the next two resellers $n-1$ and $n-2$ with varying path length n, when U is uniformly distributed on $[0, 10]$. It is observed that an upstream node always earns more than a downstream node does, and the expected payoff of a node falls with increasing n, meaning that a node earns less when the client is further away from the access point.

When U is normally distributed, one can analyze numerically to check if the economic properties still hold as when U is uniformly distributed. Figure 7(c), 8(b) and 9(b) give associated graphical plots when U is normally distributed with mean $\mu = 5$ and standard deviation $\sigma = 1.67$. Notice that in Figure 8(b), the curves for the different path length n no longer coincide. Figure 9(b) shows that the previous claims about expected payoff of nodes still hold in this example.

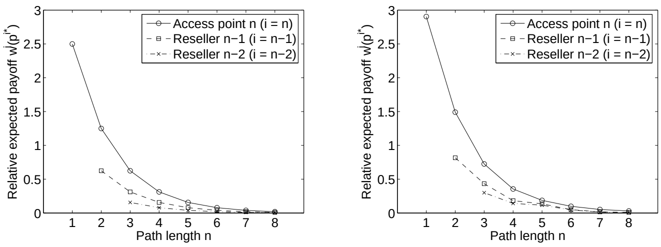
C. The Issue on Network Scaling

In the previous section, we present the analysis of pricing dynamics in the multi-hop case when uniform and normal distributions of U are assumed. Here, we investigate the scalability issue of the described multi-hop wireless network. By scalability, we mean the ability of pricing as an incentive



(a) U uniformly distributed (b) U normally distributed

Fig. 8. PBE prices in cases of different path lengths



(a) U uniformly distributed (b) U normally distributed

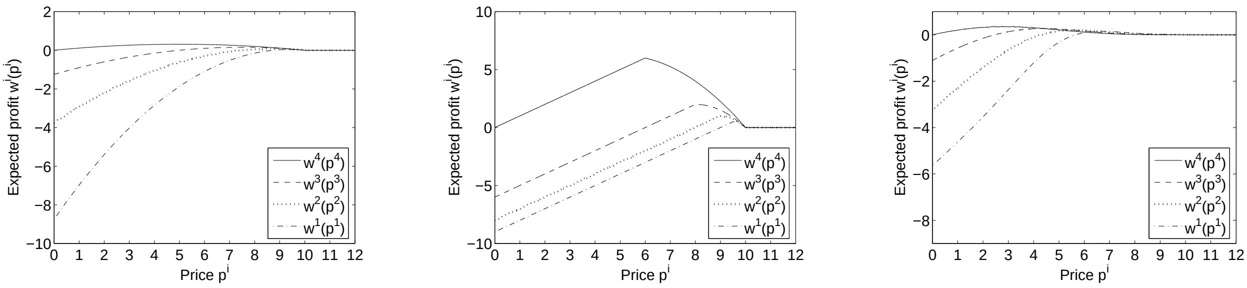
Fig. 9. Relative expected payoff of nodes with varying path length

system to encourage nodes to participate and grow the mesh network in a multi-hop fashion. The quantities concerned are the probability that a client, at an arbitrary distance, accepts her access price and connects the network, as well as the population distribution of clients. Let n_e be the radius (in terms of number of hops) of the effective multi-hop wireless network established by the access point, such that a client located more than n_e hops away from the access point will connect with a probability less than a threshold H. In the case that U is uniformly distributed, from Eq. (8), n_e can be derived as:

$$n_e = \lfloor \log_2 \frac{b}{H(b-a)} \rfloor.$$

For example, with U uniformly distributed on $[0, 10]$, a threshold of $H = 0.5$ yields $n_e = 1$ while $H = 0.1$ yields $n_e = 3$. The former case tells that the probability of a successful connection will fall below half for clients at more than one hop away; while the latter tells that the probability even falls below ten percent for clients at more than three hops away! This small effective network radius is not surprising as Eq. (8) reveals the probability that a client connects to the network decreases exponentially with path length n, unless the condition for a boundary case is satisfied. Note that the boundary case condition can be re-written as $n \geq \log_2(b/(b-a))$. Thus when a is close to b, meaning that the access point and resellers are relatively sure about client's per-slot utility, the effective network radius is large. In Figure 10, we show the probability that a client is willing to connect ($P(U \geq p^{1^*})$) against different n, the length of the network path. From the figure, one can observe that a similar conclusion can be drawn when U is normally distributed.

Though the probability that a client who is n-hop away from the access point and is willing to connect to the network



(a) U uniformly distributed—normal case (b) U uniformly distributed—boundary case (c) U normally distributed

Fig. 7. A measure of expected profit per slot per client ($w^i(p^i)$) vs. price p^i for different nodes in the four-hop case

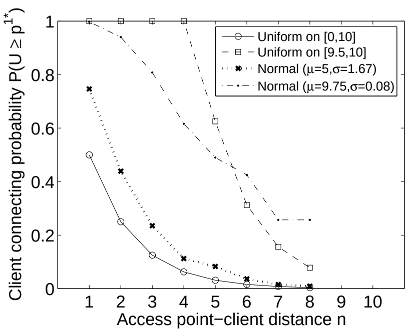


Fig. 10. Probability that a client who is n-hop away and is willing to connect under different distributions of U

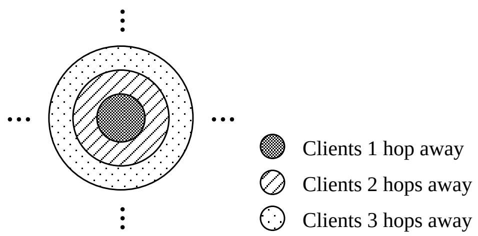


Fig. 11. A simple client population model

decreases with increasing n, should the access point and resellers think that they can still make a profit with distant clients since a larger value of n implies a larger coverage of potential clients? One can show that this argument is *not true*. Consider a simple client population model, in which clients are evenly populated geographically, the arrival rate of clients in a particular region is proportional to its area, and the wireless signal results in a mesh network where clients at different hop distances locate in areas formed by concentric circular boundaries, as in Figure 11, with the access point at the center. The following expression approximates the arrival rate of clients, who are n-hop away the access point and accept price p^{1^*}, when U is uniformly distributed and constitutes a normal case:

$$L(n) = (\pi n^2 - \pi(n-1)^2)\lambda \cdot P(U \geq p^{1^*}) = \frac{2n-1}{2^n}\left(\frac{b\pi\lambda}{b-a}\right),$$

where λ is the arrival rate of clients per unit area. The connection rate as a function of hop distance, $L(n)$, tends to 0 as n approaches infinity, hence shows that the intuition is incorrect.

The poor scaling performance of the pricing mechanism is related to the tree-like topology of the network under analysis. The multi-hop case of the unlimited capacity model describes a situation in which the client has a single route toward the access point. This constitutes a monopoly market between each node on the path and its downstream hop. The monopoly market gives each upstream node the price setting ability to maximize her profit and this introduces economic inefficiency. With pricing competition, economic scalability improves. Consider the network in Figure 12(a). The client has two paths toward the access point. Reseller A and B are under pricing competition against each other. A brief look at the case shows that the two resellers will lower their prices for the client until their costs to provide the connection service are reached, which are identical and equal to the price that the access point charges them. Thus, the access point effectively sees the client as a first-hop client and the client enjoys a price reduced to the normal first-hop price. The pricing structure of this network can be obtained by a simple extension of the one-hop case in a tree-like network. The network in Figure 12(b) shows similar properties. All resellers except the central one are under pricing competition. Only the central reseller has an opportunity to mark up the price from her upstream. The access point thus effectively sees the client as a second-hop client and the client enjoys a reduced price. This network degenerates to the two-hop case in a tree-like network. Similar arguments can be applied to the case in which a client n-hop away has more than one path to the access point. The important lesson of the above analysis is that economic scalability of a wireless mesh network is linked with the density of nodes. Sparseness introduces a large number of monopolized links in clients' selectable paths to the access point, resulting in *low economic scalability*; denseness introduces pricing competitions among nodes, apparently results in higher economic scalability. Moreover, our results in pricing structure in tree-like networks may serve as building blocks for the general pricing structure in networks of other topologies. In particular, nodes under pricing competition may not have an opportunity to mark up the price from the upstream. After taking out those nodes, the pricing structure may resemble that in a tree-like network. Pricing and scaling for networks with pricing

(a) Network A (b) Network B

Fig. 12. Two networks with pricing competition involved

competition involved remain as our future work.

Remark: In a multi-hop wireless mesh network in which the network has an unlimited capacity, clients will be charged at a fixed rate. However, if the network is sparse and each client has only one single route to the access point, the access price will not be affordable to most distant clients. It is concluded that sparseness of nodes in a wireless mesh network reduces its economic scalability.

IV. Limited Capacity Model

The formulation of the unlimited capacity model relies on the assumption that the wireless network channel and the access point's uplink have an unlimited capacity, or the access point provides no bandwidth guarantee to clients. In this section, we consider a more realistic scenario and inspect the pricing and purchasing strategies of nodes in wireless networks with a limited network capacity. Similar to the previous section, we begin with a one-hop network and show why the previous one-hop case PBE is not applicable under this new setting. A substitute for the access service provision model named "*fixed-rate, non-interrupted service*" is hence proposed and we provide an algorithm to obtain the optimal strategy of the access point in its defined strategy space. The analysis is finished with an extension to the multi-hop case, and some observations on the network scaling issue of networks with a limited capacity.

A. One-hop Case

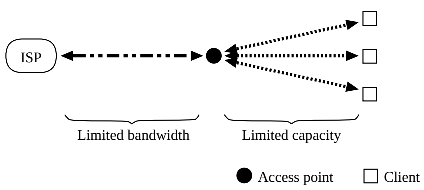


Fig. 13. Network diagram with channel capacity for the one-hop case

Here, we first present the necessary modifications to the original unlimited capacity model and transform it into the limited capacity version. The one-hop case of the limited capacity model still describes a wireless network consisting of an access point, plus clients who reach the access point directly. The distinction between the two models is that the wireless network and the access point's uplink here have a limited capacity, and the access point has to assure clients that they will have a certain amount of dedicated bandwidth, which is the premise of clients having the bandwidth independent web browsing utility function. This imposes the access point a bandwidth constraint on its profit maximization problem. Figure 13 depicts this scenario. In our model, we limit the access point to admit at most $m > 0$ clients at a time. The capacity m is a design knob of the access point. She has to evaluate stochastically the bandwidth demand of clients and the effect of multiplexing clients' traffic so as to set the right m to provide to clients the bandwidth guarantee. Any client who arrives at the access point not being immediately served due to this capacity limit will be dropped.

Another addition to the original model is an explicit client arrival process at the access point. This is necessary as the interactions between the access point and a client are now complicated by the removal of the unlimited capacity assumption—they cannot be summarized by one simple two-player game; the access point must decide its strategy on each occasion, based on its system condition at the time, such as the remaining capacity for admission. We model the client arrival behavior using a Poisson input process with a finite population of clients. Each client arrives with a rate λ at the access point, and there is a total of M clients in the population.

The last modification to the unlimited capacity model is to transform it from a discrete-slot process into a continuous-time process so as to ease our analysis when matched with the client arrival model. In the continuous-time version, the access point charges a particular client a price per unit time, or rate, $p(t)$ at time t. The variables T, τ and U are converted, in the continuous-time sense, to represent the amount of time the client connects, the amount of time the client intends to connect, and the client's utility of the service per unit time respectively. The continuous-time web browsing utility function of the client thus remains the same as its previous form of $F(T, \tau) = U \cdot \min(T, \tau)$. The access point still only knows the probability distributions of U and τ. Here, we further assume that the access point takes τ to be exponentially distributed with mean $1/\mu$.[4] Our formulation of the limited capacity model is now complete. It should be clear to see the correspondence between our model and the classical M/M/m/m/M queuing system [19].

Let us give a simple scenario to show that under the limited capacity model, the access point, on some occasions, will choose either to charge clients with *a variable rate*, or to deliberately disconnect clients, rather than adopting a fixed-rate, non-interrupting strategy for the one-hop case PBE with unlimited capacity model proposed in [3].

Lemma *1:* A fixed-rate, non-interrupting strategy is not at all time optimal to the access point under the limited capacity

[4]This assumption allows us to conduct a "mean analysis" of the system. The analysis can be extended and made more realistic by using techniques such as the "Method of Stages".

model.

Proof: Consider the case that a new client arrives at the access point, when it is at its full capacity. Let p_0 be the price of one of the m connected clients is paying. The access point may announce a price $p_0{}' > p_0$ to the new client. If the new client accepts, the access point's best response is to disconnect the old client paying p_0 and admit the new one, unless the old client accepts a raise of price from p_0 to $p_0{}'$. Thus, a fixed-rate, non-interrupting strategy is not the best response of the access point. ∎

Although the access point wishes to cease service to clients or increase the price over time to obtain higher profit, it is reasonable to believe that clients will be discouraged from buying such a kind of service since it is unrealistic to require clients to monitor the varying price continuously. Thus, we investigate a service model named the "*fixed-rate, non-interrupted service*", which is more likely to be adopted in practice.

The fixed-rate, non-interrupted service model requires a contract to be enforced between the access point and a particular client as follows:

- The access point provides connection service to the client until the client voluntarily disconnects;
- The client pays a fixed rate p for the service. The total payment is p times the duration of the service.

Note that the access point is still allowed to announce different "fixed rates" (or prices) to different clients under this scheme, but once announced, this fixed rate cannot be changed during the course of service for a particular client.

The fixed-rate, non-interrupted service contract can be enforced in numerous ways. A common approach is to establish a third party contract enforcement agent in the system. Clients will report to the enforcement agent on misbehavior of the access point, seeking the offender to be punished. The penalty is to be set heavy enough such that the access point will not go for short-term benefit of disconnecting clients or raising their rates. Another approach is to distribute standardized software to nodes participating in the wireless mesh network, which conforms to the service contract. The owner of the access point will not have the knowledge to tweak the software so as to avoid contract enforcement, just like that most Internet users will not change the transmission control protocol (TCP) in their operating systems so as to obtain a higher throughput.

Compared to the pricing model in [3], the fixed-rate, non-interrupted service model has the disadvantage that it requires contract enforcement, i.e. it is not self-enforcing. There is a wide variety of pricing models which are not self-enforcing. Among them, we pick and investigate the fixed-rate, non-interrupted service model for two reasons. First, the fixed-rate, non-interrupted service model is driven by the fair expectation of customers on Internet access service, and is one in its category which deviates the least from the original self-enforcing pricing model. This implies that there will be minimal incentives for the access point to break the service contract and use a floating rate, which is against the will of customers. For comparison, consider a pricing model in a multi-hop network which requires revenue be split evenly among the access point and the relaying nodes. In such a model, it is tempting for the nodes to break the contract for a higher profit, as how the revenue is shared has nothing to do with customer satisfaction. To enforce this contract, the overhead will be high, while for the fixed-rate, non-interrupted service model, it is not. Secondly, the slight deviation of the fixed-rate, non-interrupted service model from the original version also means that it is a harder problem to solve in comparison to those which put even more restrictions on the pricing scheme, such as requiring a common fixed rate to all clients. With the methodology to analyze the fixed-rate, non-interrupted service model, we can also solve simpler problem instances using the same approach.

A strategy of the access point under the fixed-rate, non-interrupted service model involves setting the charging rate to clients who want to be connected. The access point can make her decision based on a single parameter, namely, the number of connected clients in the system. Adopting queuing system notations, the number of connected clients in the system is represented by the current "state". For the M/M/m/m/M queuing system, it has $m+1$ states, from state 0 to state m. At state k, for all $k \in \{0, \ldots, m-1\}$, the access point has to decide the rate p_k to charge the next "to-be-admitted" client. No decision has to be made at state m as the access point is at its full capacity. Thus, a policy of the access point is completely characterized by the price or rate vector $\vec{p} = (p_0, p_1, \ldots, p_{m-1})$.

With the fixed-rate, non-interrupted service contract, we see that clients will play the following strategy to maximize her payoff, (a) connect the access point iff $U \geq p$, (b) disconnect from the access point at time $t = \tau$, with the assumption that clients with $U \geq p$, utility per unit time not less than charged rate, will not deliberately reject the first presented rate and wait until she receives a lower rate at a later time when the access point is less congested. Also, for clients rejecting the first presented rate, our Poisson client arrival process may not accurately model their possible behavior of re-probing the access rate afterward.

We now derive an expression of the expected profit per unit time, or the gain, of the access point in the long run as a function of the rate vector $\vec{p}$. The general equilibrium solution for birth-death queuing systems [19] is employed. Note that a transition from state k to state $k+1$, for all $k \in \{0, \ldots, m-1\}$, requires not only an arrival of a client, but also her willingness to accept the charged price p_k, therefore, the "arrival rate" of our model is different from that of the conventional M/M/m/m/M queuing system by a factor of $P(U \geq p_k)$ for each state k, $k \in \{0, \ldots, m-1\}$. The transition rates of our model are:

$$\begin{aligned} \lambda_k &= \begin{cases} \lambda(M-k)P(U \geq p_k) & k < m \\ 0 & \text{otherwise} \end{cases} \\ \mu_k &= k\mu \qquad\qquad\qquad\qquad\qquad k = 1, 2, \ldots, m. \end{aligned}$$

With π_k denoting the limiting probability that the system is in state k, for all $k \in \{0, \ldots, m\}$, we have

$$\pi_k = \frac{\binom{M}{k}\left(\frac{\lambda}{\mu}\right)^k \prod_{i=0}^{k-1} P(U \geq p_i)}{\sum_{j=0}^{m}\left(\binom{M}{j}\left(\frac{\lambda}{\mu}\right)^j \prod_{i=0}^{j-1} P(U \geq p_i)\right)},$$

where empty product is unity by convention. For simplification, consider that the access point earns an expected profit of p_k/μ immediately when a client connects at state k.[5] Hence, the gain of the access point is

$$
\begin{aligned}
G(\vec{p}) &= \sum_{k=0}^{m-1} \pi_k \lambda_k \left(\frac{p_k}{\mu}\right) \\
&= \frac{\sum_{k=0}^{m-1}\left[p_k(M-k)\binom{M}{k}\left(\frac{\lambda}{\mu}\right)^{k+1}\prod_{i=0}^{k} P(U \geq p_i)\right]}{\sum_{k=0}^{m}\left[\binom{M}{k}\left(\frac{\lambda}{\mu}\right)^{k}\prod_{i=0}^{k-1} P(U \geq p_i)\right]}
\end{aligned} \tag{9}
$$

The optimal policy of the access point can be obtained by maximizing Eq.(9) over the rate vector $\vec{p}$. However, using classical optimization techniques to derive a closed-form solution of the optimal policy requires solving simultaneous non-linear equations, which is complicated. Instead, we use the *policy-iteration method* in the Markovian decision theory [20] to determine the pricing for the above optimization problem.

The policy-iteration method is given in Algorithm 2. It involves an iteration cycle of two parts: the *policy-improvement routine* and the *value-determination operation*. It uses the notation g to denote the gain of the system, and introduces a set of relative values v_k, for all $k \in \{0, 1, \ldots, m\}$, which has the physical meaning of which $v_i - v_j$ is the increase in the gain caused by starting the system in state i rather than in state j. The algorithm is started in the policy-improvement routine with all relative values v_k set to 0.

The policy-improvement routine is to improve the current policy by considering alternatives in each state, based on the relative values v_k, either set to 0 initially, or obtained in the value-determination operation for the current policy $\vec{p}$. It requires solving a separable optimization problem where the design variables are $p_0, p_1, \ldots, p_{m-1}$, as shown in Algorithm 2. The solution to this optimization then forms a new policy. If the difference between this new policy and the previous policy in the iteration cycle is smaller than a pre-defined threshold, the iteration process has converged and the (near-)optimal policy is found. Otherwise, the algorithm goes into the value-determination operation and the new policy is evaluated.

The value-determination operation evaluates a policy $\vec{p}$ generated by the policy-improvement routine. It requires solving a set of equations, given in Algorithm 2, for g and all relative values v_k by setting v_m to zero. With the solution, the algorithm loops back into the policy-improvement routine.

It is worth noticing that when the client's utility rate U has a uniform distribution on the interval $[a, b]$, there exists a closed-form solution for the optimization problem in the policy-improvement routine, i.e. the optimal rate $p_k{}^*$ for each state k. As an optimal state rate $p_k{}^*$ must lie on the interval $[a, b]$, we may substitute $P(U \geq p_k)$ with $(b - p_k)/(b - a)$. Differentiation followed by root finding yields the optimal $p_k{}^*$

Algorithm 2 The policy-iteration method

Require: $\lambda > 0$, $\mu > 0$, $m > 0$ and $M > 0$

Ensure: optimal pricing policy $\vec{p}$

$\vec{p} \Leftarrow$ arbitrary value
for all k such that $0 \leq k \leq m$ **do**
  $v_k \Leftarrow 0$
end for
loop
  $\vec{q} \Leftarrow \vec{p}$
  $\{$The policy-improvement routine$\}$
  solve the following optimization problem with design variables $p_0, p_1, \ldots, p_{m-1}$:

$$
\begin{aligned}
&\max \lambda M P(U \geq p_0) v_1 - \lambda M P(U \geq p_0) v_0 + \lambda M P(U \geq p_0)\left(\frac{p_0}{\mu}\right), \\
&\max \lambda(M-k)P(U \geq p_k)v_{k+1} - (\lambda(M-k)P(U \geq p_k) + \\
&\qquad k\mu)v_k + k\mu v_{k-1}\lambda(M-k)P(U \geq p_k)\left(\frac{p_k}{\mu}\right) \\
&\qquad \text{for } k = 1, \ldots, m-1.
\end{aligned}
$$

  if not the first iteration and $\vec{q} - \vec{p} <$ predefined threshold **then**
    return $\vec{p}$
  end if
  $\{$The value-determination operation$\}$
  $v_m \Leftarrow 0$
  solve the following set of equations for $v_0, v_1, \ldots, v_{m-1}$ and g:

$$
\begin{aligned}
&g = \lambda M P(U \geq p_0) v_1 - \lambda M P(U \geq p_0) v_0 + \lambda M P(U \geq p_0)\left(\frac{p_0}{\mu}\right), \\
&g = \lambda(M-k)P(U \geq p_k)v_{k+1} - (\lambda(M-k)P(U \geq p_k) + k\mu)v_k \\
&\qquad + k\mu v_{k-1} + \lambda(M-k)P(U \geq p_k)\left(\frac{p_k}{\mu}\right) \text{ for } k = 1, ..., m-1, \\
&g = -m\mu v_m + m\mu v_{m-1}.
\end{aligned}
$$

end loop

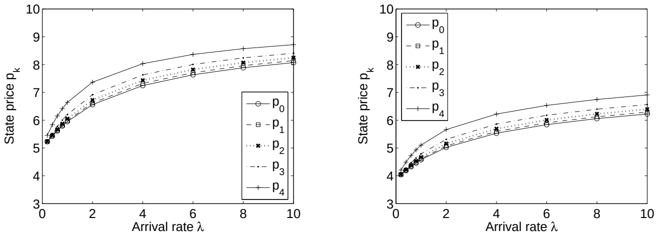
for each state k:

$$
p_k{}^* = \frac{b - \mu(v_{k+1} - v_k)}{2} \qquad k = 0, 1, \ldots, m-1.
$$

The policy-iteration method reduces the profit maximization problem of the access point to solving sets of $m+1$ simultaneous linear equations in the value-determination operation, and sets of m independent one-dimensional optimization problems in the policy-improvement routine. The computational complexity is reduced (as compared with the standard numerical optimization method), and it is shown in [20] that the above procedures guarantee the convergence to the best policy.

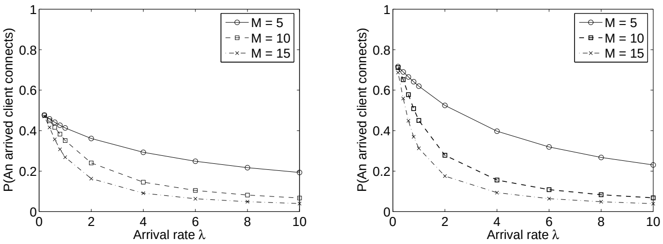
Here we show some numerical results obtained using the policy-iteration method. Cases in which client's utility rate U is uniformly distributed on $[0, 10]$, or normally distributed with a mean of 5 and a standard deviation of 1.67 are studied. We illustrate the state rates given by the policy-iteration method when the access point can support $m = 5$ clients, and there are totally $M = 10$ potential clients who want to receive the connection service. We fix the departure rate of client μ to 1 and vary the arrival rate λ from 0.2 to 10. Figure 14 shows the results. It can be observed that the state-dependent price

[5]This simplification helps reduce the state space of the model and is an approximation of the original problem. An exact solution can be obtained by including in the state space the information on the rate each client in the system is paying, and the use of "in-state reward" instead of the immediate expected profit "state transition reward" [20].



(a) U uniformly distributed (b) U normally distributed

Fig. 14. State-dependent price with varying arrival rate



(a) U uniformly distributed (b) U normally distributed

Fig. 15. Probability that a client is willing to connect with varying arrival rate and different client population sizes

rises with the number of clients in the access point system: $p_i \geq p_j$, for all $i > j$. This agrees with the economic sense that when the remaining resource, or supply, of service decreases, the price increases. Also, the state-dependent price rises with increasing λ, and this is logical as the arrival rate λ represents demand. Figure 15 gives the probability that a client is willing to accept the offered price and connects to the system. It is given by the following expression:

$$P(\text{An arrived client connects}) = \sum_{k=0}^{m-1} \pi_k P(U \geq p_k).$$

The setting is the same as in the previous case, except that we repeat with different client population, setting M to 5, 10 and 15. We see that the probability drops with the arrival rate. In addition, its value is always lower for a larger population. The result agrees with the intuition that with higher demand for the service, the probability for a successful purchase drops.

Lastly, we find in our experiments that the policy-iteration method takes on average four iterations to converge to the optimal pricing policy, for various problem size m (the number of state prices to be determined) from 1 to 100. The convergence condition is that policies in two consecutive iterations differ by less than 0.001 for every state price. A detailed evaluation is documented in our technical report [18]. The algorithm proves to be efficient in our access point profit maximization problem.

B. Multi-hop Case

We now extend the limited capacity model to the multi-hop case. We make the assumption that the bandwidth bottleneck is at the wireless channel one-hop around the access point, or at the access point's uplink to the Internet, where traffic from

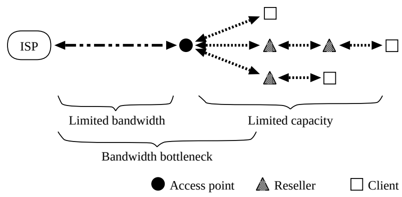


Fig. 16. Network diagram with channel capacity for the multi-hop case

all clients in the wireless mesh network merges. Hence, any reseller who has purchased Internet access service from her upstream will have adequate bandwidth for her downstream. This situation is depicted in Figure 16. In comparison with the multi-hop case in the unlimited capacity model, we see that the bandwidth constraint only affects the access point; for the resellers, their strategies only depend on the prices their respective upstream hops charge them. Thus, any node apart from the access point will follow her strategy in the unlimited capacity model here.

The focus of the multi-hop case is to devise the optimal pricing strategy of the access point, which involves determining the respective optimal prices for clients from different distances at each state. Thus, for an access point with capacity m, and with the assumption that the most distant clients arriving at the access point are from n hops away, a policy of the access point can be characterized by the price matrix $\mathbf{P} = [p_{ki}]$, $k \in \{0, \ldots, m-1\}$, $i \in \{1, \ldots, n\}$, in which p_{ki} represents the price at state k for a client i-hop away.

To ease analysis, we modify the client arrival process by removing the feature of finite client population. This is necessary, as an arrival process with finite population requires keeping track of the numbers of admitted clients at different distances, which highly complicates state information. We roll back to an arrival model originated from the M/M/m/m queuing system [19]. Assuming the most distant clients arrive from a distance of n hops, we use an arrival rate vector $\vec{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ to denote the arrival rates of clients from different distances, in which λ_i denotes the arrival rate of clients i-hop away. With μ denoting the departure rate of a client in the system, we have the following state transition rates:

$$\begin{aligned} \lambda_k &= \begin{cases} \sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{ki})) & k < m \\ 0 & \text{otherwise} \end{cases} \\ \mu_k &= k\mu \qquad\qquad\qquad\qquad k = 1, 2, \ldots, m. \end{aligned}$$

The factor moderating λ_i, the pure arrival rate of clients i-hop away, is now $P(U \geq m^i(p_{ki}))$, which is a simple reflection from the multi-hop case in the unlimited capacity model that any state rate p_{ki} charged by the access point will be marked up by all $i-1$ downstream resellers, as expressed by the function $m^i(p_{ki})$ in Eq. (4).

Further taking the simplification that the access point earns an expected profit of p_{ki}/μ immediately when a client i-hop away connects at state k[6], we can again use the policy-iteration method to solve for the optimal pricing policy of the access

[6]This simplification is again an approximation technique to reduce the state space as in the one-hop case.

point, but with the following changes. The set of equations to be solved in the value-determination operation is updated as:

$$g = \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{0i}))\right] v_1 - \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{0i}))\right] v_0 + \sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{0i}))(\frac{p_{0i}}{\mu}),$$

$$g = \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{ki}))\right] v_{k+1} - \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{ki})) + k\mu\right] v_k + k\mu v_{k-1} + \sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{ki}))(\frac{p_{ki}}{\mu}) \quad \text{for } k = 1, \ldots, m-1,$$

$$g = -m\mu v_m + m\mu v_{m-1}.$$

The set of optimization problems in the policy-improvement routine is updated as:

$$\max \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{0i}))\right] v_1 - \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{0i}))\right] v_0 + \sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{0i}))(\frac{p_{0i}}{\mu}),$$

$$\max \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{ki}))\right] v_{k+1} - \left[\sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{ki})) + k\mu\right] v_k + k\mu v_{k-1} + \sum_{i=1}^{n} \lambda_i P(U \geq m^i(p_{ki}))(\frac{p_{ki}}{\mu}) \text{ for } k = 1, .., m-1$$

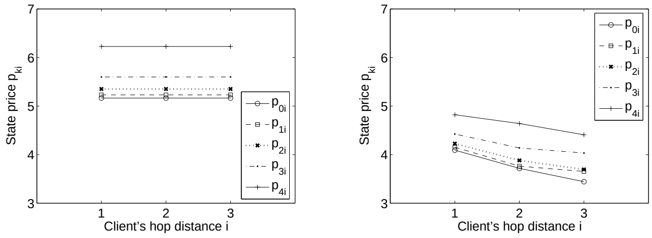
where the design variables are now p_{ki}, $k \in \{0, \ldots, m-1\}$, $i \in \{1, \ldots, n\}$. When client's utility rate U has a uniform distribution on the interval $[a, b]$, we again has a closed-form solution for the optimal rate $p_{ki}{}^*$ for clients i-hop away for each state k:

$$p_{ki}{}^* = \frac{b - \mu(v_{k+1} - v_k)}{2} \quad k = 0, .., m-1 \text{ and } i = 1, .., n.$$

One can observed that when U has a uniform distribution, optimal prices for clients at different distances are the same for each state.

The following shows some numerical results of the multi-hop case obtained by using the policy-iteration method. Figure 17 essentially plots the resulting optimal price matrices for two cases. The first case has the client's utility rate U uniformly distributed on $[0, 10]$; the second case has U normally distributed with a mean of 5 and a standard deviation of 1.67. For both cases, the arrival rate vector $\vec{\lambda}$ is $(4, 2, 1)$, the departure rate μ is 1, and the capacity of the access point m is 5. It can be shown that when U is uniformly distributed, the optimal prices for clients at different distances at each state are identical; while when U is normally distributed, the access point tends to charge a lower price for clients further away. As in the one-hop case, prices rise with the number of admitted clients.

When U is normally distributed, prices for distant clients are lower than prices for clients closer to the access point; however, the prices tend to converge when the arrival rate



(a) U uniformly distributed (b) U normally distributed

Fig. 17. Price matrix with different utility rate distributions

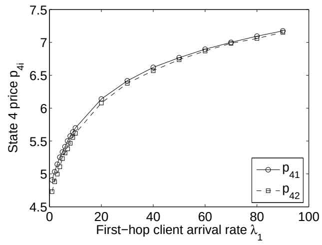


Fig. 18. Prices converge with increasing arrival rate of first-hop clients

of proximate clients increases. This is illustrated in Figure 18. The arrival rate of first-hop clients increases from 1 to 90. The remaining clients are from two hops away and arrive with a rate of 10. Departure rate μ is fixed at 1. The access point has a capacity m of 5. U is normally distributed with a mean of 5 and a standard deviation of 1.67. It can be observed that prices for first-hop and second-hop clients at state 4 converge with increasing λ_1.

C. Scalability of Networks with a Limited Capacity

In the limited capacity model, we derive the optimal pricing policy of the access point using the policy-iteration method, which provides numerical results but not closed-form state prices. Though the lack of closed-form results hinders us from analytically deducing the scalability of networks with a limited capacity, we can observe that when the access point has a capacity constraint to fulfill, the access (state) prices she sets are always higher than those when she has an unlimited capacity. In short, as the network capacity increases, the state prices the access point sets decrease, and approach the flat rate in a network with an unlimited capacity. Hence, we can conclude that the scalability of a network with a limited capacity is upper bound by the scalability of a network of the same topology but with an unlimited capacity.

Remark: In a wireless mesh network with a limited capacity, clients will not be charged at a fixed rate without contract enforcement. With the "fixed-rate, non-interrupted service" contract, the access point will charge according to the amount of remaining network capacity. The "state price" can be obtained by the efficient policy-iteration method and is found that it grows with a decrease in remaining admission quota. The scalability of a network with a limited capacity is always lower than that of a network of the same topology but with an unlimited capacity.

V. Conclusion

We have conducted a mathematical analysis of the economic behavior of nodes in a wireless mesh network, when they are making a decision to establish an Internet connection service. Two scenarios are investigated: either the network has an unlimited or limited channel capacity. First we present specific examples of the one-hop case of the unlimited capacity model, with various distributions of client's per-slot utility. We then extend the analysis to the multi-hop case and show that the price of the access service grows quickly as the service path length increases. The implication is that it becomes unaffordable for distant clients (i.e., many hops away from the access point) and the wireless mesh network may not be economically scalable. In the limited capacity case, we have proved that a fixed-rate pricing scheme similar to the one proposed in [3] is not optimal, or economically beneficial, to the access point. We further investigate a more practical "fixed-rate, non-interrupted service" model for charging. To determine the optimal price for this charging scheme, we model the problem as a Markovian decision process and use the efficient policy-iteration method to solve for the optimal pricing strategy of the access point. Numerical results show that the state price follows with supply and demand, and the economic scalability of a network with a limited capacity is lower than one with an unlimited capacity.

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Appendix

A Proof of the Strategy Profile for Multi-hop Wireless Mesh Network being a PBE

To show the strategy profile for multi-hop wireless mesh network is a PBE, we first need the following lemma.

Lemma 2: *There exists a function $p^{i^*}(p^{i+1})$, for all $i \in \{1, \ldots, n-1\}$ that satisfies properties of Eq. (2) and (3).*

Proof: The proof is generic for all $i \in \{1, \ldots, n-1\}$. Define

$$y_{p^{i+1}}(p^i) \triangleq (p^i - p^{i+1}) P(U \geq m^i(p^i)).$$

We find $p^{i^*}(p^{i+1})$ by construction. Set

$$p^{i^*}(p^{i+1}) = \min \left(\arg\max_{p^i} y_{p^{i+1}}(p^i) \right),$$

which agrees with property (2). It remains to show $p^{i^*}(p^{i+1})$ is monotonically non-decreasing. We prove this by contradiction. Suppose $p^{i^*}(p^{i+1})$ is not monotonically non-decreasing. Then there exists $({p^{i+1}}^l, {p^{i+1}}^h) : {p^{i+1}}^l < {p^{i+1}}^h$ with $p^{i^*}({p^{i+1}}^h) < p^{i^*}({p^{i+1}}^l)$. For convenience, define ${p^i}^l \triangleq p^{i^*}({p^{i+1}}^h)$ and ${p^i}^h \triangleq p^{i^*}({p^{i+1}}^l)$ so that ${p^i}^l < {p^i}^h$. For ${p^i}^h$ to be the lowest valued maximizer of $y_{{p^{i+1}}^l}(p^i)$, it is necessary that

$$({p^i}^h - {p^{i+1}}^l) P(U \geq m^i({p^i}^h)) > ({p^i}^l - {p^{i+1}}^l) P(U \geq m^i({p^i}^l)). \tag{10}$$

For ${p^i}^l$ to be the lowest valued maximizer of $y_{{p^{i+1}}^h}(p^i)$, it is necessary that

$$({p^i}^l - {p^{i+1}}^h) P(U \geq m^i({p^i}^l)) \geq ({p^i}^h - {p^{i+1}}^h) P(U \geq m^i({p^i}^h)). \tag{11}$$

Combining (10) and (11), we have

$$\frac{{p^i}^l - {p^{i+1}}^h}{{p^i}^h - {p^{i+1}}^h} > \frac{{p^i}^l - {p^{i+1}}^l}{{p^i}^h - {p^{i+1}}^l}. \tag{12}$$

But Eq. (12) implies ${p^i}^h({p^{i+1}}^h - {p^{i+1}}^l) < {p^i}^l({p^{i+1}}^h - {p^{i+1}}^l)$, which is a contradiction. Thus the $p^{i^*}(p^{i+1})$ we construct is monotonically non-decreasing, and we have found a function which satisfies properties (2) and (3). ■

Theorem 2: *The strategy profile for multi-hop wireless mesh network is a perfect Bayesian equilibrium.*

Proof: First we prove that access point n's strategy is optimal given that all other players play their respective strategies in the PBE. At the beginning of time slot 1, the expected payoff of access point n adopting a price sequence $\{p_t^n\}$ is

$$J_1^n(\{p_t^n\}) = \sum_{t=1}^{\infty} p_t^n P(U \geq \max_{u\in\{1,\ldots,t\}} m^n(p_u^n))P(\tau \geq t). \tag{13}$$

In the notation $J_1^n(\{p_t^n\})$, the superscript n denotes the node index of access point n, and the subscript 1 denotes the time slot index at which the expected payoff is evaluated. Access point n's objective is to choose a price sequence $\{p_t^n\}$ which maximizes Eq. (13). We see that such sequence must be a non-decreasing sequence. The reason is two-fold. First, the function $m^n(p^n)$ is monotonically non-decreasing, since the price mark-up functions $p^{i^*}(p^{i+1})$ for all reseller i, $i \in \{1,\ldots,n-1\}$, are monotonically non-decreasing. Hence, for any sequence $\{\tilde{p_t^n}\}$ in which there exists a u such that $\tilde{p_u^n} < \tilde{p_{u-1}^n}$, we can define a new non-decreasing sequence $\{p_t^n\}$ with $p_t^n = \max(\tilde{p_1^n},\ldots,\tilde{p_t^n})$ and we would have $J_1^n(\{p_t^n\}) > J_1^n(\{\tilde{p_t^n}\})$. Define S^+ be the set of non-decreasing price sequences. The expected payoff of access point n under non-decreasing price sequences can be expressed as

$$J_1^n|_{\{p_t^n\}\in S^+}(\{p_t^n\}) = \sum_{t=1}^{\infty} p_t^n P(U \geq m^n(p_t^n))P(\tau \geq t). \tag{14}$$

Each term in the summation of Eq. (14) is a function of a different price p_t^n, so the entire sum can be maximized by independently maximizing each term. Thus, the optimal strategy of access point n at the start of time slot 1 is to choose each p_t^n in the set $\arg\max_{p^n}[p^n P(U \geq m^n(p^n))]$ and form a price sequence $\{p_t^n\} \in S^+$.

It remains to show that access point n's strategy is optimal in any continuation game beginning at an arbitrary time slot s. At slot s, access point n chooses a subsequent price sequence $\{p_t^n\}_{t=s}^{\infty}$ to maximize her expected payoff from slot s onward:

$$J_s^n(\{p_t^n\}_{t=s}^{\infty}) = \sum_{t=s}^{\infty} \Big[p_t^n P(U \geq \max_{u\in\{s,\ldots,t\}} m^n(p_u^n)|U \geq m^n(p_{s-1}^n)) \\ P(\tau \geq t|\tau \geq s)\Big].$$

For any price sequence $\{\tilde{p_t^n}\}_{t=s}^{\infty}$ which has prices less than p_{s-1}^n, $J_s^n(\{\max(\tilde{p_t^n}, p_{s-1}^n)\}_{t=s}^{\infty}) \geq J_s^n(\{\tilde{p_t^n}\}_{t=s}^{\infty})$. Thus access point n should select on-going slot prices no less than p_{s-1}^n. Assuming $p_u^n \geq p_{s-1}^n$ for all $u \geq s$, we may write

$$J_s^n(\{p_t^n\}_{t=s}^{\infty}) = \frac{1}{P(U \geq m^n(p_{s-1}^n))P(\tau \geq s)} \\ \sum_{t=s}^{\infty} \Big[p_t^n P(U \geq \max_{u\in\{s,\ldots,t\}} m^n(p_u^n))P(\tau \geq t)\Big] \tag{15}$$

Eq. (15) has a structure that parallels to Eq. (13), with the exception of scaling factor

$$\frac{1}{P(U \geq m^n(p_{s-1}^n))P(\tau \geq s)},$$

which is unrelated to the optimality of prices chosen for slot s onward. Thus the argument which is used to show that a non-decreasing price sequence with elements chosen in the set $\arg\max_{p^n}[p^n P(U \geq m^n(p^n))]$ maximizes Eq. (13), can be re-used here to show such sequence also maximizes Eq. (15). Thus it is proved that access point n's strategy remains optimal in any continuation game.

We now prove the strategy of reseller i is the best response against the PBE strategies of other players. The proof is generic for all $i \in \{1,\ldots,n-1\}$, and is similar to that for access point n. At the beginning of time slot 1, reseller i wishes to choose a price sequence $\{p_t^i\}$ to maximize her expected payoff:

$$J_1^i(\{p_t^i\};\{p_t^{i+1}\}) = \sum_{t=1}^{\infty} \Big[(p_t^i - p_t^{i+1}) \\ P(U \geq \max_{u\in\{1,\ldots,t\}} m^i(p_u^i))P(\tau \geq t)\Big] \tag{16}$$

which is also dependent on the price sequence $\{p_t^{i+1}\}$ her upstream reseller $i+1$ charges her. Consider a modified objective:

$$\tilde{J_1^i}(\{p_t^i\};\{p_t^{i+1}\}) \triangleq \sum_{t=1}^{\infty} (p_t^i - p_t^{i+1})P(U \geq m^i(p_t^i))P(\tau \geq t). \tag{17}$$

We see that the function $p^{i^*}(p^{i+1})$, which satisfies property (2), maximizes Eq. (17) because the sum is separable and can be maximized term by term. Further check the relationship between the original objective and the modified one:

$$J_1^i(\{p_t^i\};\{p_t^{i+1}\}) \leq \tilde{J_1^i}(\{p_t^i\};\{p_t^{i+1}\}) \tag{18}$$

and

$$J_1^i|_{\{p_t^i\}\in S^+}(\{p_t^i\};\{p_t^{i+1}\}) = \tilde{J_1^i}|_{\{p_t^i\}\in S^+}(\{p_t^i\};\{p_t^{i+1}\}), \tag{19}$$

where S^+ is the set of non-decreasing price sequences. Eq. (18) and (19) imply that a non-decreasing price sequence maximizing $\tilde{J_1^i}(\{p_t^i\};\{p_t^{i+1}\})$ would also maximize $J_1^i(\{p_t^i\};\{p_t^{i+1}\})$. As access point n charges a non-decreasing price sequence, and each upstream reseller i, for all $i \in \{i+1,\ldots,n-1\}$, uses a monotonically non-decreasing price mark-up function, the price sequence $\{p_t^{i+1}\}$ received by reseller i is a non-decreasing sequence. Thus the function $p^{i^*}(p^{i+1})$, which satisfies property (3), yields a non-decreasing price sequence $\{p_t^i\}$, maximizing the original expected payoff Eq. (16).

Next we show that reseller i's strategy remains the best response in all continuation games. At an arbitrary slot s, reseller i's expected payoff from slot s onward with a price sequence $\{p_t^i\}_{t=s}^{\infty}$ is:

$$J_s^i(\{p_t^i\}_{t=s}^{\infty};\{p_t^{i+1}\}_{t=s}^{\infty}) = \sum_{t=s}^{\infty} \Big[(p_t^i - p_t^{i+1})P(U \geq \\ \max_{u\in\{s,\ldots,t\}} m^i(p_u^i)|U \geq m^i(p_{s-1}^i))P(\tau \geq t|\tau \geq s)\Big]. \tag{20}$$

Eq. (20) suggests that reseller i should only consider charging prices no less than p_{s-1}^i onward. Assuming $p_u^i \geq p_{s-1}^i$ for all

$u \geq s$, we may write

$$J_s^i(\{p_t^i\}_{t=s}^{\infty}; \{p_t^{i+1}\}_{t=s}^{\infty}) = \frac{1}{P(U \geq m^i(p_{s-1}^i))P(\tau \geq s)}$$
$$\sum_{t=s}^{\infty} \Big[(p_t^i - p_t^{i+1})P(U \geq \max_{u \in \{s,\dots,t\}} m^i(p_u^i))P(\tau \geq t)\Big]. \quad (21)$$

Eq. (21) has a structure that parallels to Eq. (16) with the exception of scaling factor

$$\frac{1}{P(U \geq m^i(p_{s-1}^i))P(\tau \geq s)},$$

which is unrelated to the optimality of prices chosen from slot s onward. Thus the argument used to show the PBE strategy of reseller i is optimal at slot 1 can be re-used here to show it remains optimal in any continuation game.

Lastly, the client receives a non-decreasing price sequence $\{p_t^1\}$, resulted from access point n's choice of non-decreasing prices, and monotonically non-decreasing price mark-up functions of all resellers. Hence, her best response at any time of the game is the myopic strategy. ∎