ENGG 2440A / ESTR 2004: Discrete Mathematics for Engineers
The Chinese University of Hong Kong, Fall 2018 — Practice Midterm Exam 3

1. Underline and explain the mistake in the following "proof."

   Theorem. Every graph has a vertex of even degree.

   Proof. By induction on the number of vertices n. When $n = 1$ the graph has a vertex of degree zero, which is even. Now assume it is true for graphs with n vertices. Let G be a graph with $n + 1$ vertices. Remove any vertex from G. By inductive hypothesis the remaining graph G' has a vertex v of even degree. Since v is also a vertex of G, G has a vertex of even degree. $\square$

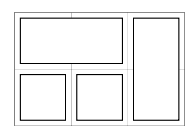
2. Prove that for every integer n there exists an integer k such that $|n^2 - 5k| \leq 1$.

3. Alice has infinitely many \$6, \$10, and \$15 stamps. Can she make all integer postages above \$30?

4. Bob has 32 *b*lue, 33 *r*ed, and 34 *g*reen balls. At every turn he takes out two balls and replaces them with two different balls by the rule below. Can he obtain 99 balls all of the same color?

   replacement rule: $bg \to rr \quad gr \to bb \quad rb \to gg \quad rr \to bg \quad bb \to gr \quad gg \to rb$

5. Use induction to show that for every $n \geq 1$, the $(n + 1) \times n$ grid can be tiled using *two* sets of the following tiles: $1 \times 1, 1 \times 2, \ldots, 1 \times n$. (See example $n = 2$.)



6. Find a stable matching for these preferences and show that there is no other stable matching.