1. Is the following deduction rule valid?

$$\frac{\forall x \exists y \colon P(x,y) \qquad \exists x \forall y \colon P(x,y)}{\forall x \forall y \colon P(x,y)}$$

2. Prove that if $m^2 + n^2$ is even then $m + n$ is even.

3. Show there exists a Die Hard scenario with three jugs and a 1 litre target in which Bruce dies if he can only use any two out of the three jugs to measure, but he survives if he uses all three jugs.

4. A knight jumps around an infinite chessboard. Owing to injury it can only make the moves shown in the diagram. Can it ever reach the square immediately to the left of its initial one?
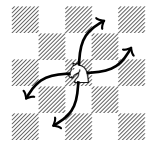
5. The vertices of a graph are the integers from 101 to 200 and their cube roots (200 in total). The pair $\{x, y\}$ is an edge if (and only if) $x + y$ is irrational. Does the graph have a perfect matching?

6. You are given a graph with 9 men and 9 women as vertices and all possible 81 man-woman pairs as edges. Let Ξ be any matching in this graph. Remove the edges in Ξ (but not the vertices.) Show that the remaining graph has a perfect matching.