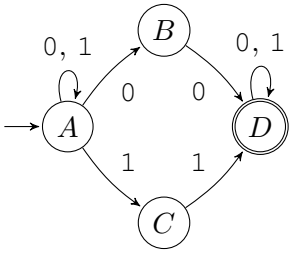
Week 3 Tutorial Session

Tutorial exercises include more problems than what a typical student can solve in 15–20 minutes. Don't be discouraged if you cannot solve all the problems within the time limit.

1. (a) Write down a regular expression for the following NFA. For this problem, you do not have to go through the procedure described in class.

   (b) Convert the following NFA into a DFA.



2. Let L be any language. We say that two strings x and y are *indistinguishable by* L if for every string z, we have $xz \in L$ if and only if $yz \in L$.

   (a) For concreteness, consider $L_1 = \{x \in \{\texttt{0},\texttt{1}\}^* \mid \text{the number of } \texttt{1}\text{'s in } x \text{ is divisible by } 3\}$. Prove that `1` and `1111` are indistinguishable by L_1.

   (b) Continuing with (a), which strings are indistinguishable from the string `1` by L_1? The set of all such strings is the *equivalence class* of the string `1` and will be denoted by $[\texttt{1}]$.

   (c) Find a string s not in $[\texttt{1}]$. What is the equivalence class of s? (We will denote this equivalence class by $[s]$)

   (d) Can you find another string t not in $[\texttt{1}]$ or $[s]$? What is the equivalence class of t?

   (e) Can you find yet another string u not in these equivalence classes?

   (f) Design a DFA for the language L_1. How are states in your DFA related to the equivalence classes?

3. (a) Write down the definition of regular expressions over an alphabet Σ.

   (b) Write down the definition of regular languages over an alphabet Σ.

   (c) Given a string w, define w^R as the string w in reverse order. That is, if $w = w_1 w_2 \dots w_n$, then $w^R = w_n w_{n-1} \dots w_1$. For example, if $w = \texttt{live}$, then $w^R = \texttt{evil}$. Given a language L, define its reversal L^R as the set of strings in L in reverse. More precisely, $L^R = \{w^R \mid w \in L\}$. For example, if $L = \{\texttt{live}, \texttt{raw}, \texttt{level}\}$, then $L^R = \{\texttt{evil}, \texttt{war}, \texttt{level}\}$.

   If L is a regular language, prove that L^R as also regular.