CSCI 3130: Formal Languages and Automata Theory — Homework 1
The Chinese University of Hong Kong, Fall 2019 — due 11:59pm Thursday September 19

Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (40 points) Give a DFA for the following languages, specified by a transition diagram. For each one of them, give a short and clear description of how the machine works. The alphabet is $\Sigma = \{\texttt{0}, \texttt{1}\}$ unless otherwise specified:

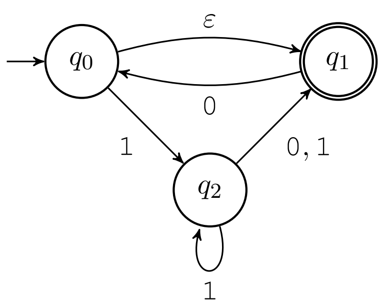
(a) $L_1 = \{w \mid w$ has least two `1`s and at most one `0`$\}$.

(b) $L_2 = \{w \mid$ the sum of digits of w is divisible by $4\}$.

(c) L_3 is the language described by $\texttt{2}^*\texttt{1}^*\texttt{0}^*$. The alphabet is $\Sigma = \{0, 1, 2\}$.

(d) $L_4 = \{w \mid w$ contains the substring `11` an odd number of times$\}$. Note that `111` contains two occurrences of `11`.

(2) (10 points) Convert the following NFA to a DFA using the method described in class. Specify the DFA by its transition diagram. The alphabet is $\Sigma = \{\texttt{0}, \texttt{1}\}$.

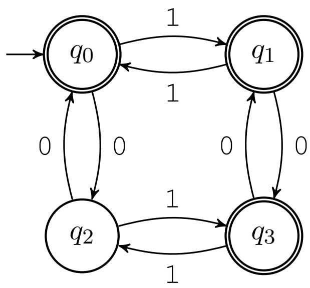


(3) (25 points) If w is a string, we say that a string x is an *initial part* of w if $w = xy$ for some string y. For example, `b` and `bcd` are both initial parts of `bcde`. Given a language L, define $L^I = \{x \mid x$ is an initial part of some $w \in L\}$. That is, L^I contains the initial parts of strings in L.

Prove that if L is a regular language, then so is L^I.

Hint: Regular language is defined *recursively*. If the desired result is true for simpler regular languages, can you show that it is also true for more complex regular languages?

(4) (25 points) Consider the following DFA:



(a) What strings stop at q_0? At q_1? At q_2? At q_3?

(b) State an induction hypothesis that will allow you to prove your answer in (a).

(c) What is the language of the DFA?