Week 4 Tutorial Session

1. For any integer $k \geq 0$, define $L_k = \{ww \mid w \in \{\texttt{0}, \texttt{1}\}^k\}$.
   (a) Write down all strings in L_3.
   (b) Prove that any DFA for L_k has at least 2^k states.
   Hint: After reading the first half of the input, what should the DFA remember? Can you come up with a set of 2^k strings that are pairwise distinguishable by L_k?

2. Let L be any language. We say that two strings x and y are *indistinguishable by L* if for every string z, we have $xz \in L$ if and only if $yz \in L$.
   (a) For concreteness, consider $L_1 = \{x \in \{\texttt{0}, \texttt{1}\}^* \mid \text{the number of } \texttt{1}\text{'s in } x \text{ is divisible by } 3\}$. Prove that 1 and 1111 are indistinguishable by L_1.
   (b) Continuing with (a), which strings are indistinguishable from the string 1 by L_1? The set of all such strings is the *equivalence class* of the string 1 and will be denoted by [1].
   (c) Find a string s not in [1]. What is the equivalence class of s? (We will denote this equivalence class by $[s]$)
   (d) Can you find another string t not in [1] or $[s]$? What is the equivalence class of t?
   (e) Can you find yet another string u not in these equivalence classes?
   (f) Design a DFA for the language L_1. How are the states in your DFA related to the equivalence classes?