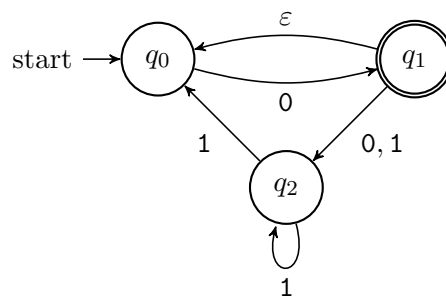


Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

- (1) (40 points) Give a DFA for the following languages, specified by a transition diagram. For each one of them, give a short and clear description of how the machine works. The alphabet is $\Sigma = \{0, 1, 2\}$:
 - (a) $L_1 = \{w \mid w \text{ has no more 1s than 0s and at most two 0s}\}$.
 - (b) $L_2 = \{w \mid \text{the sum of digits of } w \text{ is divisible by 4}\}$.
 - (c) L_3 is the language described by $(0 + 2)^*(1 + 2)^*(0 + 1)^*$.
 - (d) $L_4 = \{w \mid w \text{ contains the pattern } 11 \text{ an even number of times}\}$. Note that 111 contains two occurrences of 11.
- (2) (10 points) Convert the following NFA to a DFA using the method described in class. Specify the DFA by its transition diagram. The alphabet is $\Sigma = \{0, 1\}$.



- (3) (25 points) If w is a string, we say that a string x is an *ending* of w if $w = yx$ for some string y . For example, bcd and d are both endings of abcd. Given a language L , define $L^E = \{x \mid x \text{ is an ending of some } w \in L\}$. That is, L^E contains the endings of strings in L .

Prove that if L is a regular language, then so is L^E .

Hint: Regular language is defined *recursively*. If the desired result is true for simpler regular languages, can you show that it is also true for more complex regular languages?

- (4) (25 points) In this problem you will design an NFA that checks if an input string correctly represents a non-negative decimal number, say in a spreadsheet application. Examples of valid decimal strings:

123
12,345

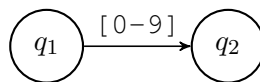
Examples of invalid decimal strings:

012
,123,456

For simplicity, assume the input string consist only of digits ‘0’ through ‘9’, and the symbol ‘,’ (comma).

- (a) (10 points) Give your definition of valid decimal strings. There is no single correct answer to this question. If you look up “decimal mark” on Wikipedia, you will find that different cultures have different digit grouping conventions. For this question, you can choose the convention you like. This question tests your ability to clearly state your definition. If you do not state your definition, you will automatically get zero points for the remaining parts as well.
- (b) (10 points) Draw an NFA that accepts valid decimal strings (and nothing else).
- (c) (5 points) Explain how your NFA works in 3–6 sentences.

When drawing the transition diagram of your NFA, you can use the shorthand notation $[0-9]$ to describe transitions labeled by all the digits ‘0’, ‘1’, ... ‘9’. An example of such a shorthand is



Similarly, you can use the notation $[1-9]$ to describe the transitions labeled by all the non-zero digits.

This is a design problem, and part of your job is to figure out a way to distinguish among valid and invalid decimal strings. There may not be a single right answer. You must describe your assumptions and reasoning clearly in your solution. Solutions that only provide a picture of an NFA with no explanation will get no credit.