Guided Complete Search for Nurse Rostering Problems

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Abstract

Nurse rostering problem is one of the most difficult scheduling problems in artificial intelligence and operation research. In general, it consists of cardinality constraints and special pattern constraints that correspond to the given workforce demands, which form a complex problem structure. Many heuristics algorithms have been proposed to solve this particular problem. In this paper, we demonstrate the efficiency of our newly defined GCS/Simplex solver, which incorporates Simplex method into the GCS framework, on some difficult nurse rostering problem instances. Experimental results show that the GCS/Simplex solver is efficient in solving this kind of scheduling problems in terms of both computation time and number of fails.

1. Introduction

Solver collaboration is an important topic in constraint satisfaction problems (CSPs) [3] research. Recently, it draws attention in combining different solvers yields a more efficient hybrid solver for CSPs. In our previous work, Guided Complete Search (GCS) has been formulated for solving hard CSP instances [2]. The main idea of GCS is that the operations of a tree search based solver are coordinated with those of another solver, which maintain the soundness and completeness of the whole hybrid scheme. The value commitments made by the tree search based solver, as well as other information such as the results of constraint propagation, also help the secondary solver to reduce the problem size in order to speed up the whole search process. In this paper, we instantiate the GCS with the Simplex method [4], obtaining a new GCS/Simplex hybrid solver. The Simplex method is an efficient iterative algorithm to solve linear programming, which achieves low average case complexity. The problem domain handled by Simplex is in the real domain, and the constraints are linear. Therefore, we need to translate the original CSP into a linear programming problem for the Simplex in order to enable collaboration between these two different types of solvers. We apply the GCS/Simplex solver to the Nurse Rostering Problem (NRP) to demonstrate the efficiency of this solver combination. The NRP consists of cardinality constraints and some of the user-defined constraints for prohibiting a specific shift pattern to appear. The search space of NRP is usually large and the problem structure is complex. Hence the problem becomes extremely difficult to solve by a general CSP solver when domain-specific heuristics is not given. The problem is computational intractable. Various approaches have been proposed to solve the problem, such as [5]. GCS/Simplex is a general CSP solver. The notable results indicate that GCS/Simplex is able to guide search towards a solution, especially for the problem consisting *alldifferent* and *cardinality* constraint. Experimental evidence shows that the GCS/Simplex solver is efficient in solving NRPs in terms of both computation time and number of fails.

2. GCS/Simplex

In a nutshell, GCS is a hybrid scheme for combining two solvers, a primary and a secondary solver. It coordinates the collaboration of the primary complete tree search-based (TS) solver and a secondary collaborating solver in order to produce a complete and efficient CSP solver. Each solver approaches solutions in its own way. Therefore, insightful information can be discovered in different aspects as that the two solvers can help each other. From the viewpoint of TS, the secondary solver acts as an “oracle” that generates heuristics for value ordering. Meanwhile, TS narrows the search space for the collaborating solver by constraint propagation after each value commitment.

The information exchange operation improves the performance for both solvers in the framework. As a result, a more efficient hybrid solver can be obtained. We employ the Simplex method as a secondary solver in GCS framework to obtain a GCS/Simplex solver. The Simplex method is an iterative procedure for linear programming problem, solving a system of linear equations in each of its steps, and stopping when either the optimum or solution infeasibility is reached. For solving an integer problem, linear relaxation is needed in order to apply Simplex method, which is a common approach for mixed integer programming (MIP) problems. With the same principle, we need to maintain a linear relaxation model of the original CSP in GCS/Simplex, namely 0-1 Linear Programming (LP) model. In short, the problem transformation is to map each value in a variable domain of the CSP into a corresponding real variable ranging over [0,1] in the LP model, and each constraint in a CSP has to be translated into a linear form in the LP model. Besides, the two models are connected by some additional constraints in order to synchronize the search space information during search.

3. Experiment on the NRPs

Nurse Rostering is the task of deciding when each nurse should report to work each day in order to satisfy the predicted workforce demand in a hospital ward. This demand may vary from shift to shift and from day to day. Besides fulfilling the demand, a rostering system must also ensure that each nurse is assigned enough work per week and that each nurse should get adequate rest between shifts. The specifications of addressed problem vary from hospitals and the user requirements, but the goal is similar as to determine the working roster for each nurse or medical specialist in hospital. In general, it consists of cardinality constraints and special pattern constraints that correspond to the given workforce demands. Solving nurse rostering problem is well-known difficult in AI research, since the problem structure is complex and the demands are tight.

A GCS/Simplex solver has been implemented with an ILOG Solver 6.0 and a COIN-OR LP solver (CLP). Experiments are conducted on the selected test cases from [5], which are shown extremely difficult to solve by a general CSP solver. In the experiment, the GCS/Simplex solver is superior to both ILOG solver and the MIP solver in terms of computation time and number of fails. ILOG Solver cannot find a solution or proof the problem unsatisfiability within the time limit in this experiment; while the GCS/Simplex solver solves all problem instances in less than a second, which outperforms the MIP solver an order of magnitude.

4. Concluding remarks

This paper outlines an idea of the newly defined GCS/Simplex solver, which incorporates Simplex method into the guided complete search framework. In order to apply Simplex method, a CSP has to be transformed into a linear form. We have a transformation method for obtaining a 0-1 linear programming model, which enables the GCS/Simplex to solve problems consist of *alldifferent* and *cardinality* constraint efficiently. With the linear programming model, we observed that a special pattern occurs in a linear relaxed solution given by the Simplex method, and which can be used to determine a better value order for guiding the primary tree search solver in GCS.

To evaluate the performance, we apply the GCS/Simplex to solve a well-known difficult nurse rostering problem. Experimental evidence shows that the guidance provided by the Simplex method is promising and directs the search toward a solution for this kind of problems. Furthermore, the results show that GCS/Simplex is able to solve certain hard problem instances without specific prior design or domain knowledge, which outperforms a tree search based CSP solver and a mixed integer programming solver standalone execution.

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