CSCI3160: Special Exercise Set 10

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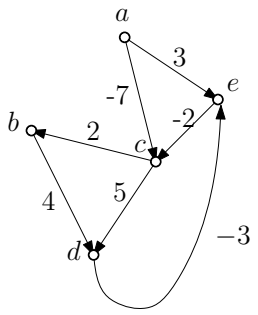
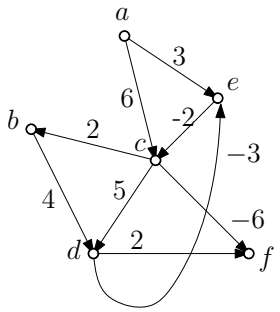
Problem 1. Consider the weighted directed graph $G = (V, E)$ below.



Note that there are negative cycles in the graph. Attempt the following tasks:

- Run Bellman-Ford's algorithm (which performs 4 rounds of edge relaxations on the above input) using a as the source vertex, and show the $dist(v)$ values for every $v \in V$.
- Based on the $dist(v)$ values computed in the previous task, explain how we can determine the existence of negative cycles in time proportional to $|E|$.

Problem 2. Consider the weighted directed graph $G = (V, E)$ below.



Suppose that we run Johnson's algorithm on G. Recall that the algorithm re-weights all the edges to make sure that every edge should carry a non-negative weight. Give the weight of each edge after the re-weighting.

Problem 3 (Textbook Exercise 25.3-4). Recall that, given a weighted directed graph $G = (V, E)$, Johnson's algorithm re-weights all the edges. Prof. Goofy proposes to replace Johnson's re-weighting strategy with the following one:

- Find the smallest edge weight z in G (e.g., for the graph G shown in Problem 2, $z = -6$).
- Re-weight each edge (u, v) in G by adding $-z$ to its weight, namely, (u, v) carries the weight $w(u, v) - z$ after the re-weighting.

Let G' be the resulting graph obtained by applying Prof. Goofy's strategy. Prove: a shortest path from u to v in G' is not necessarily a shortest path from u to v in G.

Problem 4* (Textbook Exercise 25.2-6). Adapt the Floyd-Warshall algorithm to detect the presence of a negative-weight cycle in $O(|V|^3)$ time.