CSCI3160: Quiz 1

Name: Student ID

Problem 1 (40%). Let n and m be two even integers. Prove: if $m = t \cdot n + n/2$ for some integer $t \geq 1$. Prove: the GCD (greatest common divisor) algorithm we discussed in the class finds the GCD of m and n in $O(1)$ time.

Note: the algorithm in general runs in $O(\log m)$ time; your mission is to show that its running time is $O(1)$ when m and n have the relationship mentioned earlier.

Solution. $GCD(n, m) = GCD(n, n/2) = GCD(0, n/2) = n/2$. Therefore, the algorithm finishes in $O(1)$ time.

Problem 2 (20%). Consider the following set S of intervals: $S = \{[10, 70], [35, 50], [5, 15], [25, 90], [30, 40], [20, 60], [35, 80], [10, 25]\}$. Run the greedy activity selection algorithm discussed in the class on S. Indicate the intervals selected by the algorithm in the order they are picked.

Solution. First interval: $[5, 15]$; second: $[30, 40]$.

Problem 3 (40%). Suppose that A is an array of n integers that have been sorted in ascending order. Explain how to use binary search to perform the following operation in $O(\log n)$ time: given an integer q (which may not appear in A), find *how many* integers in A are smaller than or equal to q.

Solution. If $n = 0$ or 1, the answer can be trivially found in $O(1)$ time. Consider now $n \geq 2$.

- If $A[\lceil n/2 \rceil] \leq q$, recursively find the number x of integers at most q in the subarray from $A[1 + \lceil n/2 \rceil]$ to $A[n]$, and return $\lceil n/2 \rceil + x$;
- Otherwise, recursively find the number x of integers at most q in the subarray from $A[1]$ to $A[\lceil n/2 \rceil]$, and return x.