CSCI3160: Regular Exercise Set 7

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Problem 1. Let s and t be strings with lengths m and n respectively, satisfying the condition that $s[m] = t[n]$. In the lecture, we proved:

$$edit(s,t) = \min \begin{cases} edit(s[1..m-1], t[1..n-1]) \\ 1 + edit(s, t[1..n-1]) \\ 1 + edit(s[1..m-1], t). \end{cases}$$

Prove: the above result can be simplified into: $edit(s,t) = edit(s[1..m-1], t[1..n-1])$.
(Hint: you can leverage the above result in your proof.)

Problem 2*. In the class we proved the "grand lemma" only for the case where $s[m] = t[n]$. In this problem, we will cover the other case where $s[m] \neq t[n]$. Let s and t be strings with lengths m and n respectively, satisfying the condition that $s[m] \neq t[n]$. Prove:

$$edit(s,t) = \min \begin{cases} 1 + edit(s[1..m-1], t[1..n-1]) \\ 1 + edit(s, t[1..n-1]) \\ 1 + edit(s[1..m-1], t). \end{cases}$$

Problem 3. Let s be a sequence of n letters. Design an $O(n)$-time algorithm to decide whether it is possible to delete $n-6$ letters from s so that the remaining sequence of 6 letters reads "secret". For example, the answer is yes for "as**s**df**ec**fas**r**df**es**t**", but no for "assdfecfaserdfst".

Problem 4 (Longest Common Subsequence; Section 15.4 of the Textbook). Let σ and s be two strings such that $|\sigma| \leq |s|$. We call σ a *subsequence* of s if it is possible to turn s into σ by repeatedly deleting letters. For example, "hell" is a subsequence of "asdfhljeljlasfdf" but "hello" is not and neither is "hlle".

You are given two strings s, t with lengths m and n, respectively. Give an $O(mn)$-time algorithm to find a common subsequence of s and t that has the greatest length. For example, if $s =$ "algorithm" and $t =$ "logarithmic", a possible output can be "grithm".