CSCI: Special Exercise Set 3

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Problem 1. Let $f(n)$ be a function of positive integer n. We know:

$$\begin{aligned} f(1) &= 1 \\ f(2) &= 2 \\ f(n) &= 3 + f(n-2). \end{aligned}$$

Prove $f(n) = O(n)$.

Problem 2. Let $f(n)$ be a function of positive integer n. We know:

$$\begin{aligned} f(1) &= 1 \\ f(2) &= 2 \\ f(n) &= n/10 + f(n-2). \end{aligned}$$

Prove $f(n) = O(n^2)$.

Problem 3. Let $f(n)$ be a function of positive integer n. We know: We know:

$$f(1) = f(2) = ... = f(1000) = 1$$

and for $n > 1000$

$$f(n) = 5n + f(\lceil n/1.01 \rceil).$$

Prove $f(n) = O(n)$. Recall that $\lceil x \rceil$ is the ceiling operator that returns the smallest integer at least x.

Problem 4. Let $f(n)$ be a function of positive integer n. We know:

$$\begin{aligned} f(1) &= 1 \\ f(n) &= 10 + 2 \cdot f(\lceil n/8 \rceil). \end{aligned}$$

Prove $f(n) = O(n^{1/3})$.

Problem 5. Let $f(n)$ be a function of positive integer n. We know:

$$\begin{aligned} f(1) &= 1 \\ f(n) &= f(\lceil n/4 \rceil) + f(\lceil n/2 \rceil) + n. \end{aligned}$$

Prove $f(n) = O(n)$.

Problem 6. Consider a set S of n integers that are stored in an array (*not* necessarily sorted). Let e and e' be two integers in S such that e is positioned before e'. We call the pair (e, e') an *inversion* in S if $e > e'$. Write an algorithm to report all the inversions in S. Your algorithm must terminate in $O(n^2)$ time.

For example, if the array stores the sequence $(10, 15, 7, 12)$, then your algorithm should return $(10, 7)$, $(15, 7)$, and $(15, 12)$.