CSCI2100/ESTR2102: Midterm - Paper 1

Hand-write all your solutions on paper. Take a picture of the paper **together with** your CUHK student ID. Upload the picture to Blackboard or email it to the instructor at *taoyf@cse.cuhk.edu.hk*. Your must do so within 15 minutes after the quiz has started.

Problem 1. (30%) Define $f(n) = 1 + c + c^2 + c^3 + ... + c^n$ where c is a positive real number. Prove:

1. $f(n) = O(n)$ if $c = 1$;
2. $f(n) = O(c^n)$ if $c > 1$;
3. $f(n) = O(1)$ if $c < 1$.

Answer.
1. Trivial and omitted.
2. $f(n) = \frac{c^{n+1}-1}{c-1}$. It is easy to verify that $f(n) \le c^n$ for all $n \ge 1$.
3. $f(n) = \frac{1-c^{n+1}}{1-c} \le \frac{1}{1-c} = O(1)$.

Problem 2. (30%) Suppose that you are given n distinct integers in an array A. All the integers are (i) in the range $[1, 10n^2]$ and (ii) multiples of n. Describe an algorithm to sort A in $O(n)$ time.

Answer. First, decrease $A[i]$ by n for each $i \in [1, n]$. This takes $O(n)$ time. After this, all the integers A are in the range $[1, 10n]$. Then, perform counting sort on A in $O(U + n) = O(10n + n) = O(n)$ time, where U is the size of the range (which is $10n$). Finally, increase $A[i]$ by a factor of n for each $i \in [1, n]$ in $O(n)$ time. The array A at this time is the sorted order.