Basic Concepts of Graphs

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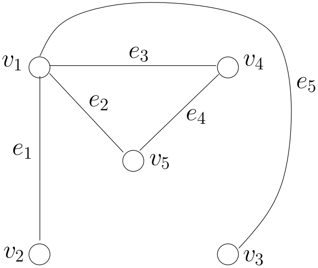
Undirected Graphs

An **undirected graph** is a pair of (V, E) where:

- V is a set of elements, each of which called a node.
- E is a set of **unordered pairs** $\{u, v\}$ where u and v are nodes.

A node is also called a **vertex**. Each element $\{u, v\} \in E$ is also called an **edge**. Node u is a **neighbor** of v; the two vertices are **adjacent** to each other.

Example



This is an undirected graph where there are 5 vertices $v_1, v_2, ..., v_5$, and 5 edges $e_1, e_2, ..., e_5$.

Directed Graphs

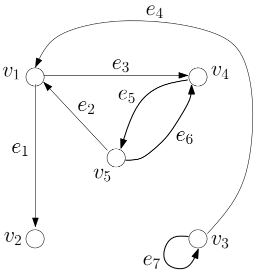
An **directed graph** is a pair of (V, E) where:

- V is a set of elements, each of which called a **node**.
- E is a set of pairs (u, v) where u and v are nodes in V.

A node is also called a **vertex**. Each element $\{u, v\} \in E$ is also called an **edge**.

Each element $(u, v) \in E$ is a **directed edge**. More specifically, it is an **outgoing** edge of u and an **incoming** edge of v. Accordingly, v is an **out-neighbor** of u and u is an **in-neighbor** of v.

Example

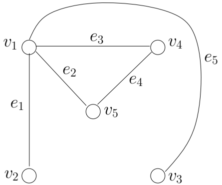


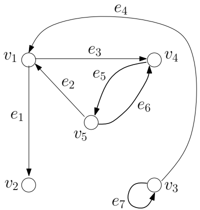
This is a directed graph (V, E) where there are 5 vertices $v_1, v_2, ..., v_5$, and 7 edges $e_1, e_2, ..., e_7$. Edge e_6, for instance, is an outgoing edge of v_5 and an incoming edge of v_4.

Degrees

- In an undirected graph, the **degree** of a vertex u is the number of edges of u.
- In a directed graph, the **out-degree** of a vertex u is the number outgoing edges of u, and its **in-degree** is the number of its incoming edges.

Example





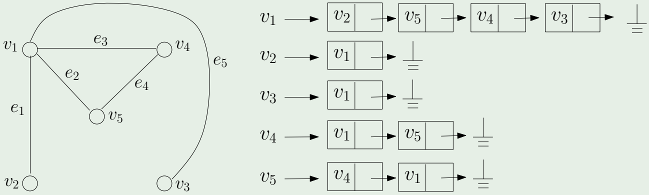
In the left graph, the degree of v_5 is 2. In the right graph, the out-degree of v_3 is 2 and its in-degree is 1.

Next, we discuss two common ways to store a graph: **adjacency list** and **adjacency matrix**. In both cases, we represent each vertex in V using a unique id in $1, 2, ..., |V|$.

Adjacency List – Undirected Graphs

Each vertex $u \in V$ is associated with a linked list that enumerates all the vertices adjacent to u.

Example

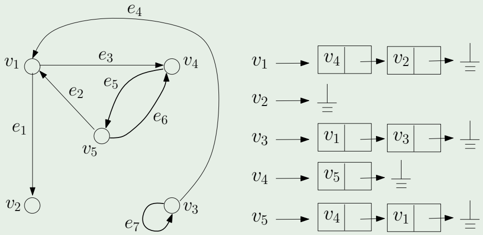


Space $= O(|V| + |E|)$.

Adjacency List – Directed Graphs

Each vertex $u \in V$ is associated with a linked list that enumerates all the out-neighbors of u.

Example

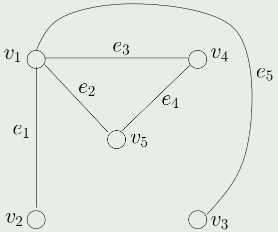


Space $= O(|V| + |E|)$.

Adjacency Matrix – Undirected Graphs

A $|V| \times |V|$ matrix A where $A[u, v] = 1$ if $(u, v) \in E$, or 0 otherwise.

Example



	v_1	v_2	v_3	v_4	v_5
v_1	0	1	1	1	1
v_2	1	0	0	0	0
v_3	1	0	0	0	0
v_4	1	0	0	0	1
v_5	1	0	0	1	0

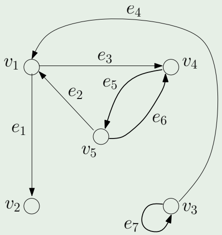
- A must be symmetric.
- Space $= O(|V|^2)$.

Think: How to store A so that, for any vertices $u, v \in V$, we can find out if they have an edge in constant time?

Adjacency Matrix – Directed Graphs

Defined in the same way as in the undirected case.

Example



	v_1	v_2	v_3	v_4	v_5
v_1	0	1	0	1	0
v_2	0	0	0	0	0
v_3	1	0	1	0	0
v_4	0	0	0	0	1
v_5	1	0	0	1	0

- A may not be symmetric.
- Space $= O(|V|^2)$.