

Exercises: Determinant

Problem 1. Calculate the determinant of the following matrix:

$$\begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$$

Problem 2. Calculate the determinant of the following matrix:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Problem 3. Calculate the determinant of the following matrix:

$$\begin{bmatrix} 0 & 4 & -6 \\ 4 & 0 & 10 \\ -6 & 10 & 0 \end{bmatrix}$$

Problem 4. Suppose that \mathbf{A} is an $n \times n$ matrix. Prove: $\det(c\mathbf{A}) = c^n \det(\mathbf{A})$.

Problem 5. Calculate

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & 0 & 0 \\ a_{41} & a_{42} & 0 & 0 \end{vmatrix}$$

Problem 6. Calculate

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & 0 & 0 & 0 \\ a_{41} & a_{42} & 0 & 0 & 0 \\ a_{51} & a_{52} & 0 & 0 & 0 \end{vmatrix}$$

Problem 7. Let \mathbf{A} be an $n \times n$ matrix. Prove:

- If we switch two columns of \mathbf{A} , $\det(\mathbf{A})$ gets multiplied by -1 .
- If we multiply a column of \mathbf{A} by a non-zero value α , $\det(\mathbf{A})$ gets multiplied by α .
- Let \mathbf{c}_i and \mathbf{c}_j be two different columns of \mathbf{A} . If we replace \mathbf{c}_i by $\mathbf{c}_i + \alpha\mathbf{c}_j$, $\det(\mathbf{A})$ remains the same.

Problem 8. Calculate

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}.$$