Exercises: Line Integral by Coordinate

Problem 1. Let C be the curve from point $p = (0, 0)$ to $q = (2, 4)$ on the parabola $y = x^2$. Calculate $\int_C (x^2 - y^2) dx$.

Problem 2. Let $\boldsymbol{r}(t) = [t, t^2, t^3]$ and $\boldsymbol{f}(x, y, z) = [x - y, y - z, z - x]$. Let C be the curve from the point of $t = 0$ to the point of $t = 1$. Calculate $\int_C \boldsymbol{f}(\boldsymbol{r}) \cdot d\boldsymbol{r}$.

Problem 3. Same as in Problem 2, except that C is defined by decreasing t from 1 to 0 (i.e., reversing the direction as in Problem 2).

Problem 4. Calculate $\int_C \boldsymbol{f}(\boldsymbol{r}) \cdot d\boldsymbol{r}$ where $\boldsymbol{f}(x, y) = [y^2, -x^2]$, and C is the arc from $(0, 0)$ to $(1, 4)$ on the curve $y = 4x^2$.

Problem 5. Calculate

$$\int_C xy\, dx + x^2 y^2\, dy$$

where C is the quarter-arc from $(1, 0)$ to $(0, 1)$ on the circle $x^2 + y^2 = 1$.

Problem 6. Let $\boldsymbol{r}(t) = [x(t), y(t)]$ where $x(t) = \cos(t)$ and $y(t) = \sin(t)$. Let p be the point given by $t = \pi/4$. Calculate $\frac{dx}{ds}$ at p.

Problem 7. Let $\boldsymbol{r}(t) = [x(t), y(t), z(t)]$. Let p be the point given by $t = t_0$. Prove that $[\frac{dx}{ds}(t_0), \frac{dy}{ds}(t_0), \frac{dz}{ds}(t_0)]$ is a unit tangent vector at p.

Problem 8. This problem allows you to see the equivalence of line integral by arc length and line integral by coordinate. Let $\boldsymbol{r}(t) = [x(t), y(t)]$ where $x(t) = \cos(t)$ and $y(t) = \sin(t)$. Convert $\int_C x\, dx + \int_C y^2\, dy$ to line integral by arc length.